Economics 3304 **Basic Research Techniques**
Isaac McFarlin **University of Texas at Dallas**

Problem Set #4:
Due on Monday, October 23rd

Note: Solutions must be legible and submitted on separate sheets of paper; that is, do not write solutions on this sheet. Furthermore, problems calling for a graphical solution are to be handwritten; graphs, which are created using a spreadsheet package, for example are not acceptable.

1. Make a graph of the PDF and the CDF, and compute $\Pr[0 \le Y \le 1]$ for each of the following cases:

   a. The PDF of Y is given by $p(y) = 0.25$ for $-1 \le y \le 3$.
      i. **P(y)=0.25y+0.25**
      ii. **P[0<=Y<=1] = (0.25*1 + 0.25) – (0.25*0 + 0.25) = 0.25**

   b. The PDF of Y is given by $p(y) = 0.25 + 0.25y$ for $0 \le y \le 2$.
      i. **P(y) = 0.25y+0.125y^2**
      ii. **P[0<=Y<=1] = 0.375**

   c. The PDF of Y is given by $p(y) = 0.5e^{-0.5y}$ for $0 \le y \le \infty$.
      i. **P(y) = 1 – e$^{-0.5y}$**
      ii. **P[0<=Y<=1] = 0.3935**

2. Say that the probability of any given UTD student gaining admission into Law School is 0.35. Suppose that this coming year 160 UTD students apply to Law School.

   a. What is the expected number of students getting into Law School?
      i. **mean=np=160*0.35=56**

   b. Using the normal approximation to the binomial distribution, what is the probability that more than 60 students will be admitted?
      i. **var=np(1-p)=160*.35*.65=36.4**
      ii. **stdev=6.03324**
      iii. **P(y > 60) = P(Y >= 60.5) because of normal approximation**
      iv. **= P(Z >= (60.5-56)/6.03324) = P(Z >= 0.746) = 0.5-0.2722 = 0.2278**

3.

   a. Given Y ~ N(0,1), compute Pr[0.6 ≤ Y ≤ ∞].
      i. **0.2743**

   b. Given Y ~ N(1,8), compute Pr[0.6 ≤ Y ≤ ∞].
      i. **P[Z >= -0.14] = 0.5557**

c. Given Y ~ N(3,12), compute Pr[2 ≤ Y ≤ 7].
   i. **P[-0.29 <= Z <= 1.16] = 0.4911**

d. Given Y ~ N(3,12), compute Pr[-2 ≤ Y ≤ 4].
   i. **P[-1.45 <= Z <= 0.29] = 0.5410**

4. Use Excel to answer the following questions:

   a. Given Y ~ $\chi^2(6)$, compute Pr[Y > 4]
      i. **0.67668**

   b. Given Y ~ t(20), compute Pr[Y > 1.89]
      i. **0.03667**

   c. Given Y ~ F(3,22), compute Pr[Y > 3.1].
      i. **0.04760**

5. You operate a convenience store. Business is random: some days, a lot of customers come by; other days, the store is quiet. In the month of March, daily sales average $2000, with a standard deviation of 600, but nothing else is known about the distribution of sales per day.

   a. Verify that you may apply the Central Limit Theorem for the question posed in part b.
      i. **31 days is "large" enough, but independence across days is perhaps questionable.**

   b. What is the probability that total sales in March exceed $67500 (and that therefore you had a pretty good month)?
      i. **P[average daily sales >= 67500/31] = P[Z >= 1.646] = 0.0499**

6. Question 3 from the course packet in chapter 14 (pg. 197-198). For this question, compute the descriptive statistics (mean and variance for both X and Y) and then compute the <u>covariance</u> and <u>correlation coefficient</u>.
   a. Mean:
      i. $\mu_x = 0(.25 + .13 + 0) + 1(.1 + .15 + .12) + 2(.05 + .1 + .1) = 0.87$

      ii. $\mu_y = 0(.25 + .1 + .05) + 1(.13 + .15 + .1) + 2(0 + .12 + .1) = 0.82$

   b. Variance:
      i. $\sigma^2_x = (.25 + .13 + 0)*(0 - 0.87)^2 + (.1 + .15 + .12)*(1 - 0.87)^2 + (.05 + .1 + .1)*(2 - 0.87)^2 = 0.6131$

      ii. $\sigma^2_y = (.25 + .1 + .05)*(0 - 0.82)^2 + (.13 + .15 + .1)*(1 - 0.82)^2 + (0 + .12 + .1)*(2 - 0.82)^2 = 0.5876$

c. Covariance:

   i. **σ_{xy} = 0.25(0-0.87)(0-0.82) + 0.1(1-0.87)(0-0.82) + 0.05(2-0.87)(0-0.82) + 0.13(0-0.87)(1-0.82) + 0.15(1-0.87)(1-0.82) + 0.1(2-0.87)(1-0.82) + 0(0-0.87)(2-0.82) + 0.12(1-0.87)(2-0.82) + 0.1(2-0.87)(2-0.82) = 0.2766**

d. Correlation Coefficient:

   i. **ρ_{xy} = 0.2766/√(0.6131)(0.5876) = 0.46084**

7. Question 13 from the course packet in chapter 14 (pg. 199).

   a. **Mean = 8.05, Standard Dev. = 3.28**

   b. **Mean = 8.5, Standard Dev. = 3.06**

   c. **Mean = 8.95, Standard Dev. = 3.28**

   d. **While we can eyeball the results from a-c and intuitively know that since the standard deviation of both stocks is equal, then the optimal allocation will be 50% investment in each stock, we can also prove this mathematically:**

      i. **Use the formula to compute the standard deviation of the portfolio.**
      ii. **Use all values as stated except set the proportion of funds to IBM = x**
      iii. **Hence, the proportion of funds for TXU = 1 - x (note that we can only do this because we are dealing with 2 stocks)**
      iv. **The formula should look like this: $\sigma^2 = 25x^2 + 25(1-x)^2 + 2(25)(-0.25)(1 - x)x$**
      v. **Next, differentiate with respect to x, set the equation to zero and then solve for the value of x.**
      vi. **This will yield x = 0.5, hence 50% of portfolio with IBM stock is optimal.**