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Correlation

1.1 Summary

1.1.1 Definitions

1. The *Pearson correlation coefficient*, denoted \( r_{W,Y} \) or \( r \), is used to determine the relationship between two variables \( W \) and \( Y \). The values of \( r_{W,Y} \) can vary from \(-1\) to \(+1\).

- A *positive* value of \( r_{W,Y} \) \((0 < r_{W,Y} \leq 1)\) indicates that the two variables \( W \) and \( Y \) vary in the same way. In other words, as \( W \) increases so does \( Y \). More formally, you would say that there is a *positive linear relationship* between the two variables.

- A *negative* value of \( r_{W,Y} \) \((-1 \leq r_{W,Y} < 0)\) indicates that the two variables \( W \) and \( Y \) vary in an opposite way (as \( W \) increases, \( Y \) decreases). More formally, you would say that there is a *negative linear relationship* between the two variables.

- A *null* value of \( r_{W,Y} \) \((r_{W,Y} = 0)\) indicates that there is no linear relationship between the two variables \( W \) and \( Y \).

2. The *squared* coefficient of correlation, \( r_{W,Y}^2 \), measures the amount of variation that is common to both \( W \) and \( Y \).

1.1.2 Steps in Performing a Correlation Analysis

1. Represent the data graphically. Draw a scatterplot of the \( W \) and \( Y \) measurements.

2. Compute \( r \). There are several ways of computing \( r \). We use the so-called *cross-product formula*:

\[
r_{W,Y} = \frac{\sum_{s=1}^{S} (W_s - M_W)(Y_s - M_Y)}{\sqrt{SS_W} \times SS_Y}
\]
1.1 Summary

The numerator in this equation is called the *cross-product* of the deviations. It is calculated by multiplying the deviation from the mean of the $W$ measurement by the deviation from the mean of the $Y$ measurement for each observation, and then taking the sum of these products. The denominator is the square root of the product of the sum of squares of $W$ and $Y$.

3. Compute $r^2$.

4. Interpret the results obtained in steps 1, 2, and 3.

1.1.3 Example of Correlation Analysis

The variable $W$ is the number of hours that a subject studied for an exam. The variable $Y$ is the grade each subject received on a scale from 0 to 5. There are 5 observations. The subscript of $S$ and $Y$ denotes the observation.

\[
\begin{align*}
W_1 &= 2 & W_2 &= 3 & W_3 &= 4 & W_4 &= 5 & W_5 &= 6 \\
Y_1 &= 1 & Y_2 &= 2 & Y_3 &= 2 & Y_4 &= 5 & Y_5 &= 5
\end{align*}
\]

1. Draw a scatterplot of the $W$ and $Y$ measurements.

2. Compute $r$.
   - Compute the mean of $W$ and $Y$.

   \[
   M_W = \frac{1}{s} \sum_{s=1}^{S} W_s = \frac{1}{5} (2 + 3 + 4 + 5 + 6) = 4.00
   \]
\[ M_Y = \frac{1}{s} \sum_{s=1}^{S} Y_s = \frac{1}{5}(1 + 2 + 2 + 5 + 5) = 3.00 \]

- Fill in the following table.

<table>
<thead>
<tr>
<th>Obs</th>
<th>W</th>
<th>( M_W )</th>
<th>( w )</th>
<th>( w^2 )</th>
<th>Y</th>
<th>( M_Y )</th>
<th>( y )</th>
<th>( y^2 )</th>
<th>( w \times y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>-2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>-2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sum )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( SS_W )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- The cross-product (\( SCP \)) is obtained by computing the sum of the right most column. Or, equivalently with a formula:

\[
SCP_{WY} = \sum_{s=1}^{S} (W_s - M_W)(Y_s - M_Y) \\
= (W_1 - M_W)(Y_1 - M_Y) + \ldots + (W_5 - M_W)(Y_5 - M_Y) \\
= (2 - 4)(1 - 3) + (3 - 4)(2 - 3) + (4 - 4)(2 - 3) + (5 - 4)(5 - 3) + (6 - 4)(5 - 3) \\
= (-2 \times -2) + (-1 \times -1) + (0 \times -1) + (1 \times 2) + (2 \times 2) \\
= (4 + 1 + 0 + 2 + 4) \\
= 11.00
\]

Recall that:

\[
SS_W = \sum_{s=1}^{S} (W_s - M_W)^2 \quad \text{and} \quad SS_Y = \sum_{s=1}^{S} (Y_s - M_Y)^2
\]

- The denominator is calculated using the values for \( SS_W = 10 \) and \( SS_Y = 14 \) from the table (i.e., the sum of the 5th and 9th
columns respectively):
\[
\sqrt{SS_W \times SS_Y} = \sqrt{10 \times 14} = \sqrt{140} = 11.8322
\]

- \( r \) is computed by dividing the cross-product by the denominator:
\[
r_{W,Y} = \frac{11.00}{11.80} = .93
\]

3. Compute \( r^2 \)
\[
r^2 = .93^2 = .86
\]

4. Interpret your results.
There is a positive linear relationship between the number of hours studied and the grade received: the more hours spent in studying, the better the grade. Specifically, the squared coefficient of correlation indicates that grades and numbers of hours spent studying share 86% of their variance. However, because the time spent studying and the grade are both dependent variables (we didn’t control them), this result does not mean that the amount of time causes the good grade. It could as well be that students with a good grade are interested in the topic, and hence, spend a lot of time studying.

1.2 Exercises

Exercise 1.1: Plotting

Given the three following sets of \( W \) and \( Y \) measurements for 10 observations:

1. Label the axes below and draw a scatterplot of each set of \( W \) and \( Y \) measurements.

Set 1  
Set 2  
Set 3
2. **What can you conclude for each set of measurements from the scatterplots?**

- **Set 1:**

- **Set 2:**

- **Set 3:**

---

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Exercise 1.2: Children, television, and aggression

A social psychologist wants to know if there is a relationship between the amount of time a child spent watching TV and the child’s score on a test of aggression. She asks the parents of a group of children from an elementary school to answer a questionnaire in order to establish the number of hours each child spends watching TV each day. Meanwhile, she administers an aggression test to the children and recorded the score of each child. The data from the questionnaire and the aggression test scores are presented in the following table (W represents the amount of hours spend watching TV and Y the aggression score).

<table>
<thead>
<tr>
<th>W</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>5</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

1. Draw a scatterplot.

2. What can you conclude?

3. Fill in Table 1.2 on the facing page.
### TABLE 1.2

The following abbreviations are used:

\[
\begin{align*}
W &= \text{Observed values of } W \\
M_W &= \text{Mean of } W \\
w &= (W - M_W) \\
w^2 &= (W - M_W)^2 \\
y &= \text{Observed values of } Y \\
M_Y &= \text{Mean of } Y \\
y^2 &= (Y - M_Y)^2 \\
w \times y &= \text{Product of adjacent values of } W \text{ and } Y \\
w^2 \times y^2 &= (W - M_W)^2 \times (Y - M_Y)^2 \\
SCP_{WY} &= \text{Sum of cross products of } W \text{ and } Y \\
SS_Y &= \text{Sum of squares of } Y \\
SS_w &= \text{Sum of squares of } W \\
\end{align*}
\]
4. Using the information in the previous table, compute the Pearson coefficient of correlation or $r_{W,Y}$.

5. Does this value of $r_{W,Y}$ appear to be in agreement with your scatterplot?

6. What is the amount of variation shared by $Y$ and $W$?

7. What should the social psychologist conclude from her study? ______

____________________________________________________________________
____________________________________________________________________
Exercise 1.3: Another simple correlation

For the following set of measurements given in Table 1.3:

<table>
<thead>
<tr>
<th>W</th>
<th>2</th>
<th>5</th>
<th>1</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

1. **Draw a scatterplot.**

2. **What can you conclude?**

3. **Fill in Table 1.4 on the following page.**

4. **Compute the Pearson coefficient of correlation** $r$. 

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The following abbreviations are used:

\[ w = (W - M_W); \]
\[ w^2 = (W - M_W)^2; \]
\[ y = (Y - M_Y); \]
\[ y^2 = (Y - M_Y)^2; \]
\[ w \times y = [(W - M_W) \times (Y - M_Y)]; \]

\[ \sum \]

\[ SS_W \]

\[ SS_Y \]

\[ SCP_{wY} \]
5. What can you conclude?

6. What is the main limitation of the Pearson coefficient of correlation?
Exercise 1.4: Faces and Words

Pinotnoir & Aligotay (1998) want to know if memory for words is related to memory for faces. So they measure for 6 subjects (randomly chosen from their pool of subjects) the number of words recognized and the number of faces recognized after learning 50 of each and a delay of 2 days. Their results are given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
<th>$S_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>W = Words</strong></td>
<td>15</td>
<td>7</td>
<td>3</td>
<td>10</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td><strong>Y = Faces</strong></td>
<td>11</td>
<td>23</td>
<td>18</td>
<td>27</td>
<td>21</td>
<td>20</td>
</tr>
</tbody>
</table>

1. Draw a scatterplot of these data.

2. What is the value of the coefficient of correlation between the number of faces recognized and the number of words recalled?
3. How much variance do they have in common?
2

Statistical Test - $F$ test

2.1 Summary

A statistical test can be used to determine if the correlation computed for a sample really exists in the population, or if it is just a fluke. If it is just a fluke then the results are not very likely to be repeated if another sample is used. We would like to know the probability that the $r$ computed from a sample could happen even though the correlation in the entire population is zero.

2.1.1 Hypotheses

- Null hypothesis - $H_0$: there is no linear relationship between the variables for the population as a whole.
- Alternative hypothesis - $H_1$: there is a linear relationship between the variables for the population as a whole.
- We reject the null hypothesis when it is shown to be unlikely (not very probable) and accept the alternative hypothesis.

2.1.2 Level of significance - $\alpha$

- Sets a precise upper limit to the notion of “unlikely.”
- Traditional levels are $\alpha = .05$ and $\alpha = .01$.
- Using $\alpha = .05$ means we are willing to reject the null hypothesis of no correlation in the population if the $r$ we computed is likely to happen only 5% of the time when, in fact, there is no linear correlation in the population (i.e., the null hypothesis is true).

The $F$ distributions

- There are many $F$ distributions.
- The shape of a particular $F$ distribution depends upon 2 parameters: $\nu_1$ and $\nu_2$. 
2.2 Summary

- \( \nu_1 \) is the degrees of freedom of correlation, and is 1.

\[
\nu_1 = df_{\text{correlation}} = 1
\]

- \( \nu_2 \) is number of degrees of freedom of error and is the number of observations minus 2.

\[
\nu_2 = df_{\text{error}} = S - 2
\]

- Using these two parameters and the desired \( \alpha \) level we locate a critical value in the \( F \) Table. This value is called \( F_{\text{critical}} \).

**Statistical index \( F \)**

We calculate an \( F \) value for the correlation as:

\[
F = \frac{r^2}{1 - r^2} \times (S - 2)
\]

**Deciding**

- We compare the calculated \( F \) to the critical value found in the table.

- If \( F \geq F_{\text{critical}} \) we reject the null hypothesis and accept the alternative.

- If \( F < F_{\text{critical}} \) we fail to reject the null hypothesis and suspend judgment.

**We risk being wrong two ways**

- Type I error.
  - Rejecting the null hypothesis when it is, in fact, true.
  - A “false alarm” (we have falsely supported the alternative hypothesis).
  - The probability of this error is \( \alpha \) - we can determine this.

- Type II error
  - Failing to reject the null hypothesis when it is false.
  - A “miss” (we have missed accepting the alternative hypothesis).
  - The probability of this error is \( \beta \) - we cannot determine this.
  - Power is \( 1 - \beta \). It is the complement of a miss.
2.2 Exercises

Exercise 2.1: Children, television, and aggression, cont'd.

This exercise is a continuation of the exercise in the chapter on Correlation. To refresh your memory, a social psychologist wants to know if there is a relationship between the amount of time a child spent watching TV and the child’s score on a test of aggression. She asks the parents of a group of children from an elementary school to answer a questionnaire in order to establish the number of hours each child spends watching TV each day. Meanwhile, she administers a test of aggression to the children and records the score of each child. The data from the questionnaire and the aggression test scores are presented in the following table (W represents the amount of hours spend watching TV and Y the aggression score).

<table>
<thead>
<tr>
<th>W</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>5</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

We found that \( r_{W,Y} \) was .94 and \( r^2 = .88 \). How reliable is this result? To answer that question we need to perform a statistical test.

1. **State the null hypothesis.**

2. **State the alternative hypothesis.**
3. **Using** $\alpha = .01$ **find the value of** $F_{\text{critical}}$ **in the** $F$ **table.**

4. **Calculate the value of the criterion** $F$.

5. **State your statistical conclusion.**

   ________________________________________
   ________________________________________
   ________________________________________
   ________________________________________
   ________________________________________
   ________________________________________
3

Regression

3.1 Summary

3.1.1 Definitions

1. The regression line is the straight line that best describes the relationship between $X$ and $Y$ measurements. It is used to predict the $Y$ measurements from the $X$ measurements. Figure 1 shows that this line is the line that is the best fit to the points in the scatterplot of $X$ and $Y$.

2. The regression equation represents the best fitting regression line for predicting $Y$ from $X$. This equation specifies the line that minimizes (i.e., keeps at a minimum value) the square of the vertical deviations of all the points in the scatterplot from the line.

3.1.2 How to find the regression line

The general form of the regression equation is

$$\hat{Y} = a + bX$$

where
3.1 Summary

- $\hat{Y}$ is the predicted value of $Y$,
- $a$ is where the line crosses the $Y$ axis, and is called the intercept (the value of $X$ is 0 at this point),
- and $b$ is the slope of the line (the ratio of the change that occurs in $Y$ to the corresponding change that occurs in $X$).
- The value for the slope $b$ is obtained by multiplying the ratio of the standard deviation of $Y$ to the standard deviation of $X$ by the correlation between $X$ and $Y$, or by dividing the $SCP_{XY}$ by $SS_X$.

$$b = \frac{\hat{\sigma}_Y}{\hat{\sigma}_X} r_{XY} = \frac{SCP_{XY}}{SS_X}$$

- The value for the intercept $a$ is obtained as:

$$a = M_Y - bM_X$$

3.1.3 Regression example

Let us go back to the example presented for manually calculating correlation and suppose that now the subjects are assigned a given number of hours of study (i.e., hours of study is now an independent variable). The results are given in the table below where $X$ is the number of hours that a subject studied for an exam, and $Y$ is the grade each subject received on a scale from 1 to 5. Note that since the variable “number of hours of study” is now an independent variable (we manipulate it) we denote it $X$ instead of $W$.

<table>
<thead>
<tr>
<th>Subject</th>
<th>$X$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$S_2$</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$S_3$</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$S_4$</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$S_5$</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

For your convenience, we kept the same data values as in the exercise on correlation. From that we know that:

1. The Pearson coefficient of correlation, $r_{X,Y}$ is equal to .93.
2. The sum of squares of $X(\text{SS}_X)$ is equal to 10.
3. The sum of squares of $Y(\text{SS}_Y)$ is equal to 14.
4. The sum of cross-products \( SCP_{XY} \) is equal to 11.

5. The mean of the \( Y \) measurements, \( M_Y \), is equal to 3.

6. The mean of the \( X \) measurements, \( M_X \) is equal to 4.

7. The total number of subjects (observations), \( S \), is equal to 5.

8. Regression equation

   (a) Computing the slope.
   To compute the slope, \( b \), we need first to compute the standard deviation of \( X \), \( \hat{\sigma}_X \), and the standard deviation of \( Y \), \( \hat{\sigma}_Y \).

   \[
   \hat{\sigma}_X = \sqrt{\frac{SS_X}{S-1}} = \sqrt{\frac{10}{4}} = 1.58
   \]

   and

   \[
   \hat{\sigma}_Y = \sqrt{\frac{SS_Y}{S-1}} = \sqrt{\frac{14}{4}} = 1.87
   \]

   The slope then is given by:

   \[
   b = \frac{\hat{\sigma}_Y}{\hat{\sigma}_X} r_{X,Y} = \frac{1.87}{1.58} \times .93 = 1.10 \quad \text{OR} \quad b = \frac{SCP_{XY}}{SS_X} = \frac{11}{10} = 1.10
   \]

   (b) Computing the intercept.
   From the slope and the means of the \( X \) and \( Y \) measurements we can find the value of the intercept:

   \[
   a = M_Y - bM_X = 3 - (1.10 \times 4) = 3 - 4.40 = -1.40
   \]

   (c) Finding the regression equation.
   To find the regression equation, it suffices now to substitute the slope and the intercept by their value in the general equation:

   \[
   \hat{Y} = a + bX = -1.40 + 1.10X
   \]

9. Predicting the \( Y \) values from the \( X \) values
   We can use the previous equation to predict the \( Y \) measurements from the \( X \) measurements:

   \[
   \hat{Y}_s = -1.40 + 1.10X \\
   \hat{Y}_1 = -1.40 + (1.10 \times 2) = 0.80 \\
   \hat{Y}_2 = -1.40 + (1.10 \times 3) = 1.90 \\
   \hat{Y}_3 = -1.40 + (1.10 \times 4) = 3.00
   \]
\[
\begin{align*}
\hat{Y}_4 &= -1.40 + (1.10 \times 5) = 4.10 \\
\hat{Y}_5 &= -1.40 + (1.10 \times 6) = 5.20
\end{align*}
\]

If we compare the predicted values \( \hat{Y} \), with the actual values of \( Y \) we can see that the predicted values are not very far from the actual values but still differ from them. How do we know if we can trust those predicted values? How do we know if the relationship between \( X \) and \( Y \) exists for the population?

### 3.1.4 Determining the quality of the prediction

The total sum of squares of the \( Y \) measurements,

\[
SS_Y = \sum_{s=1}^{S} (Y_s - M_Y)^2
\]

can be decomposed into two parts:

1. The *sum of squares of regression*, \( SS_{\text{regression}} \), which represents the deviations from the predicted scores to the mean:

\[
SS_{\text{regression}} = \sum_{s=1}^{S} (\hat{Y}_s - M_Y)^2
\]

2. The *sum of squares residual*, \( SS_{\text{residual}} \), which represents the deviations of the actual scores from the predicted scores (i.e., the scores obtained by using the regression equation):

\[
SS_{\text{residual}} = \sum_{s=1}^{S} (Y - \hat{Y}_s)^2
\]

with

\[
SS_Y = SS_{\text{regression}} + SS_{\text{residual}}
\]

From these sums of squares we can compute the regression variance and the residual variance (or variance of error). The general formula to compute a variance (also called a *mean square*) is given by:

\[
\hat{\sigma}^2_{\text{something}} = MS_{\text{something}} = \frac{SS_{\text{something}}}{df_{\text{something}}}
\]

where \( df \) represents the degrees of freedom, \( SS \) represents the sum of squares, and \( MS \) the mean square.

The number of degrees of freedom for the sum of squares of regression, \( df_{\text{regression}} \), is 1. The number of degrees of freedom for the residual sum of squares, \( df_{\text{residual}} \), is \( S - 2 \).
1. The variance or mean square of regression is computed as

\[ \hat{\sigma}^2_{\text{regression}} = MS_{\text{regression}} = \frac{SS_{\text{regression}}}{df_{\text{regression}}} = \frac{SS_{\text{regression}}}{1} = SS_{\text{regression}} \]

2. The variance of the residual or error variance is computed as:

\[ \hat{\sigma}^2_{\text{residual}} = MS_{\text{residual}} = \frac{SS_{\text{residual}}}{df_{\text{residual}}} = \frac{SS_{\text{residual}}}{S - 2} . \]

The null hypothesis \((H_0)\) stating that there is no linear relationship between \(X\) and \(Y\) can then be tested using the following \(F\) ratio:

\[ F = \frac{\hat{\sigma}^2_{\text{regression}}}{\hat{\sigma}^2_{\text{residual}}} = \frac{MS_{\text{regression}}}{MS_{\text{residual}}} \]

When the null hypothesis is true, the regression variance is nothing but another estimate of the error variance. In other words, when the null hypothesis is true, the regression variance and the residual variance are similar in value and hence the \(F\) ratio is close to 1. When the null hypothesis is not true (i.e., when there is a linear relationship between \(X\) and \(Y\)) the regression variance is greater than the error variance, and hence, the \(F\) ratio is greater than 1.

The significance of the \(F\) ratio can then be tested by finding the critical value of \(F\) for \(\nu_1 = df_{\text{regression}} = 1\) and \(\nu_2 = df_{\text{residual}} = S - 2\) degrees of freedom in a Fisher distribution table. If the calculated value of \(F\) is greater than the critical value read in the table, \(F\) is said to be significant and we can reject the null hypothesis and accept the alternative hypothesis (i.e., we can be confident that a linear relationship does exist between \(X\) and \(Y\)).

### 3.1.5 Back to the example

We can now come back to our previous example and test the null hypothesis, \(H_0\), stating that there is no relationship between the number of hours a student studies for a test and the grade she or he receives.

From the previous table, we can find the different sums of squares necessary to compute the \(F\) ratio.

- The sum of squares of regression can be read in the right most column of the table, or computed with the following formula:

\[
SS_{\text{regression}} = \sum_{s=1}^{S} (\hat{Y}_s - M_Y)^2
\]

\[= (.80 - 3)^2 + (1.90 - 3)^2 + (3 - 3)^2 + (4.10 - 5)^2 + (5.20 - 5)^2 \]
### TABLE 3.1 Some useful sums for the regression example.

<table>
<thead>
<tr>
<th>Obs</th>
<th>X</th>
<th>Y</th>
<th>$M_Y$</th>
<th>$\hat{Y}$</th>
<th>$Y - \hat{Y}$</th>
<th>$(Y - \hat{Y})^2$</th>
<th>$\hat{Y} - M_Y$</th>
<th>$(\hat{Y} - M_Y)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0.80</td>
<td>0.20</td>
<td>0.04</td>
<td>-2.20</td>
<td>4.84</td>
</tr>
<tr>
<td>$S_2$</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1.90</td>
<td>0.10</td>
<td>0.01</td>
<td>-1.10</td>
<td>1.21</td>
</tr>
<tr>
<td>$S_3$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>3.00</td>
<td>-1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$S_4$</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>4.10</td>
<td>0.90</td>
<td>0.81</td>
<td>1.10</td>
<td>1.21</td>
</tr>
<tr>
<td>$S_5$</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>5.20</td>
<td>-0.20</td>
<td>0.04</td>
<td>2.20</td>
<td>4.84</td>
</tr>
<tr>
<td>$\sum$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td>1.90</td>
<td>0.00</td>
<td>12.10</td>
</tr>
</tbody>
</table>

$$= (-2.20)^2 + (-1.10)^2 + (0)^2 + (1.10)^2 + (2.20)^2$$

$$= 4.84 + 1.21 + 0 + 1.21 + 4.84$$

$$= 12.10$$

- The sum of squares of residual can be read in the 6th column of the table, or computed with the following formula:

$$SS_{\text{residual}} = \sum_{s=1}^{S} (Y_s - \hat{Y}_s)^2$$

$$= (1 - .80)^2 + (2 - 1.90)^2 + (2 - 3.00)^2 + (5 - 4.10)^2 + (5 - 5.20)^2$$

$$= (.20)^2 + (.10)^2 + (-1.00)^2 + (.90)^2 + (-.02)^2$$

$$= .04 + .01 + 1.00 + .81 + .04$$

$$= 1.90$$

The variance (or mean square) of regression and residual can then be obtained by dividing the sum of squares by their corresponding degrees of freedom:

$$\hat{\sigma}^2_{\text{regression}} = MS_{\text{regression}} = \frac{SS_{\text{regression}}}{df_{\text{regression}}} = \frac{12.10}{1} = 12.10$$

and

$$\hat{\sigma}^2_{\text{residual}} = MS_{\text{residual}} = \frac{SS_{\text{residual}}}{df_{\text{residual}}} = \frac{1.90}{5 - 2} = .63$$

The $F$ ratio is obtained by dividing the variance of regression by the variance of error:

$$F = \frac{\hat{\sigma}^2_{\text{regression}}}{\hat{\sigma}^2_{\text{residual}}} = \frac{MS_{\text{regression}}}{MS_{\text{residual}}} = \frac{12.10}{.63} = 19.20$$

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To decide if we can reject the null hypothesis, $H_0$, we need to compare this $F$ value with the critical value of $F$ for $\nu_1 = df_{\text{regression}} = 1$ and $\nu_2 = df_{\text{residual}} = S - 2 = 3$. We can read in the Fisher distribution table that at $\alpha = .05$, $F_{\text{critical}} = 10.13$ and that at $\alpha = .01$, $F_{\text{critical}} = 34.12$. Since the computed value of $F = 19.20$ is larger than the critical value at $\alpha = .05$, we can reject the null hypothesis for this significance level and conclude that there is a linear relationship between the time spent to study for a test and the grade obtained. However, if we set our significance level at $\alpha = .01$, then the computed value of $F = 19.20$ is smaller than the critical value $F_{\text{critical}} = 34.12$. Hence, we cannot reject the null hypothesis at $\alpha = .01$ and must suspend our judgment for this significance level.
Exercise 3.1: Predicting aggression

The social psychologist that we encountered in earlier in the chapter on correlation decided to carry out further analysis of her research hypothesis. Recall that she wanted to know if there was a relationship between the amount of time a child spends watching TV and the child's score on a test of aggression. She asked the parents of 6 children to participate in an experiment. Each child was randomly assigned to a number of hours watching TV per day. After one week, she administered an aggressiveness test to the children and recorded their scores. She obtained the following data (for your convenience, the numbers are the same as in the correlation example):

<table>
<thead>
<tr>
<th>X</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>5</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

X represents the independent variable (i.e., the amount of hours spent watching TV) and Y is the dependent variable (i.e., the aggressiveness score). The psychologist found a linear positive relationship between the aggressiveness scores and the number of hours the children spend watching TV \( (r_{X,Y} = .94) \). Now, she would like to know if she can reliably predict a child's score on the aggression test from the time he or she spent watching TV.

1. Find the equation of the regression line.

(a) Knowing that the sum of squares of X and Y are 28 and 18, respectively, compute the value for the slope \( b \)
(b) Knowing that the means of $X$ and $Y$ are 4 and 4, respectively, compute the value for the intercept $a$.

(c) Substitute the value for $b$ and $a$ in the general equation.

(d) Draw the scatterplot of $X$ and $Y$, and the regression line.
2. Using the regression equation find the predicted $Y$ values ($\hat{Y}$) for all the values of $X$.

3. Determine the quality of your prediction.
   (a) What is the null hypothesis?
   (b) Fill in the work table on page on the next page
   (c) What is the sum of squares of regression?
<table>
<thead>
<tr>
<th>Obs</th>
<th>$X$</th>
<th>$Y$</th>
<th>$M_Y$</th>
<th>$\hat{Y}$</th>
<th>$Y - \hat{Y}$</th>
<th>$(Y - \hat{Y})^2$</th>
<th>$\hat{Y} - M_Y$</th>
<th>$(\hat{Y} - M_Y)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(d) What is the residual sum of squares?

(e) What is the number of degrees of freedom regression?

(f) What is the number of degrees of freedom residual?
(g) **Compute the regression mean square.**

(h) **Compute the residual mean square.**

(i) **Compute the $F$ ratio.**
3.2 Exercises

(j) What are the critical values of \( F \) at \( \alpha = .05 \) and \( \alpha = .01 \)?

4. What should the social psychologist conclude from her study? _____

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
4
Multiple Regression

4.1 Exercises

Exercise 4.1: Serving up ...

Perusing the latest issue of the *Journal of Generic and Fictional Psychology*, we came across the following study. For your information, this journal (which, as the names indicates, is a completely fictitious APA journal) specializes in reporting data without any explanations. So, feel free to supply your own design.

Georges Platdujour (2002) measures the values of the dependent variable \( Y \) on 12 subjects for which the values of the independent variables \( X \) and \( T \) are also known. The following data were obtained:

<table>
<thead>
<tr>
<th>Subject</th>
<th>( X )</th>
<th>( T )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>−1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>−1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>−1</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>10</td>
<td>−1</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>−1</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>−1</td>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>
1. What is the equation for the prediction of $Y$ from $T$ and $X$?

2. Are $T$ and $X$ orthogonal independent variables (justify your answer)?
3. What are the values of $r_{Y,X}^2$ and $r_{Y,T}^2$?

4. What is the value of $R_{Y,XT}^2$?
5. Is the prediction of $Y$ significantly better than chance?

6. What is the respective importance of each independent variable in the prediction?
5

ANOVA One factor, $S(A)$

5.1 Summary

5.1.1 Notation Review

For this example we will consider an experiment that manipulated the diet of rats to see if the type of food the rats ate had an effect on their effectiveness in running a maze. Five rats received “diet 1” and five other rats received “diet 2”. After one month on the particular diets, we recorded how long it took the rats to run the maze. We will denote the “diet” by $A$. The number of levels of the “diet” factor is $A = 2$, (“diet 1” and “diet 2” are denoted $a_1$ and $a_2$, respectively). There are 5 subjects at each level of the factor, therefore $S = 5$. Remember that $S$ is the number of subjects per experimental group. The dependent variable $Y$ is the time (in minutes) to run the maze. The score for a particular subject in a particular diet group is denoted $Y_{as}$, where $a$ can be 1 or 2 and $s$ indicates the subject (ranging from 1 to 5). So for example, the score of subject $Y_{1,2} = 14$ and the score of subject $Y_{2,4} = 8$.

<table>
<thead>
<tr>
<th>Diet</th>
<th>Time to run the maze</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 14 8 16 12</td>
</tr>
<tr>
<td>2</td>
<td>9   7 10 8 6</td>
</tr>
</tbody>
</table>

5.1.1.1 Sums

Sum of all the scores:

$$Y_\cdot = \sum_{a=1}^{A} \sum_{s=1}^{S} Y_{as} = 100$$

Sum of scores for diet 1:

$$Y_{1,} = \sum_{s=1}^{S} Y_{1s} = 60$$
5.1 Summary

Sum of scores for diet 2:

\[ Y_2 = \sum_{s=1}^{S} Y_{2s} = 40 \]

5.1.1.2 Means

**Definition 5.1.**

*Grand mean:* The sum of all the scores divided by the total number of subjects.

\[ M_\cdot = \frac{Y_\cdot}{A \times S} = \frac{100}{10} = 10.00 \]

*Group 1 mean:* The sum of the group (diet) 1 scores divided by the number of subjects in this group.

\[ M_{1.} = \frac{Y_{1.}}{S} = \frac{60}{5} = 12.00 \]

*Group 2 mean:* The sum of the group (diet) 2 scores divided by the number of subjects in this group.

\[ M_{2.} = \frac{Y_{2.}}{S} = \frac{40}{5} = 8.00 \]

5.1.1.3 Degrees of Freedom

The total number of degrees of freedom is 1 less than \((A \times S)\) (where \(A\) is the number of levels of \(A\), and \(S\) is the number of subjects at each level). With a formula we get

\[ df_{\text{total}} = (A \times S) - 1 = (2 \times 5) - 1 = 9 \]

The number of degrees of freedom between groups is 1 less than the number of levels of \(A\) \((A - 1)\). Note that the \(df_{\text{between}}\) corresponds to the \(df_{\text{regression}}\) in regression analysis. With a formula we get

\[ df_{\text{between}} = df_{\text{regression}} = df_{A} = A - 1 = 2 - 1 = 1 \]

The number of degrees of freedom within groups is found by multiplying the number of levels of \(A\) by the number of subjects \((S)\) minus 1 in each of the \(A\) levels. Note that the \(df_{\text{within}}\) corresponds to the \(df_{\text{residual}}\) in regression analysis. With a formula we get

\[ df_{\text{within}} = df_{\text{residual}} = df_{S(A)} = A \times (S - 1) = 2 \times 4 = 8 \]
5.1.1.4 Sum of Squares Total

To calculate the total sum of squares: subtract the grand mean from each score, square this difference, and take the sum of all of these squared differences.

\[
SS_{\text{total}} = \sum_{a=1}^{A} \sum_{s=1}^{S} (Y_{as} - M..)^2
\]

\[
= (10 - 10)^2 + (14 - 10)^2 + (8 - 10)^2 + \ldots + (8 - 10)^2 + (6 - 10)^2
\]

\[
= 0 + 16 + 4 + 36 + 4 + 1 + 9 + 0 + 4 + 16
\]

\[
= 90.00
\]

5.1.1.5 Sum of Squares Between

To calculate the sum of squares between: for each level of \(A\), subtract the grand mean from the mean of that level and square this difference, sum all the squared values, multiply this sum by the number of subjects per level. Note that the sum of squares between (\(SS_{\text{between}}\)) is equivalent to the sum of squares regression (\(SS_{\text{regression}}\)) in regression.

\[
SS_{\text{between}} = SS_{\text{regression}} = S \times \sum_{a=1}^{A} (M_a. - M..)^2
\]

\[
= 5 \times [(12 - 10)^2 + (8 - 10)^2] = 5 \times (4 + 4)
\]

\[
= 40.00
\]

5.1.1.6 Sum of Squares Within

To calculate the sum of squares within: for each level of \(A\), subtract the mean of that level from each score in the level, square each difference, and sum the squared values. The sum of squares residual (\(SS_{\text{residual}}\)) in regression corresponds to this sum of squares within.

\[
SS_{\text{within}} = SS_{\text{residual}} = \sum_{a=1}^{A} \sum_{s=1}^{S} (Y_{as} - M_a.)^2
\]

\[
= (10 - 12)^2 + (14 - 12)^2 + (8 - 12)^2 + \ldots + (8 - 8)^2 + (6 - 8)^2
\]

\[
= 4 + 4 + 16 + 16 + 0 + 1 + 1 + 4 + 0 + 4
\]

\[
= 50.00
\]

5.1.1.7 Mean Square Between

\[
MS_{\text{between}} = MS_A = \frac{SS_{\text{between}}}{df_{\text{between}}} = \frac{40}{1} = 40.00
\]
5.1.1.8 **Mean Square Within**

\[
MS_{\text{within}} = MS_{S(A)} = \frac{SS_{\text{within}}}{df_{\text{within}}} = \frac{50}{8} = 6.25
\]

5.1.1.9 **The F index**

\[
F_A = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{MS_A}{MS_{S(A)}} = \frac{40}{6.25} = 6.40
\]

5.1.1.10 **The ANOVA table**

We can now present our results in an ANOVA table.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between, (A)</td>
<td>1</td>
<td>40.00</td>
<td>40.00</td>
<td>6.40</td>
</tr>
<tr>
<td>Within, (S(A))</td>
<td>8</td>
<td>50.00</td>
<td>6.25</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>9</td>
<td>90.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.1.2 **Are the results significant?**

To determine if results are significant, compare the \(F\) value that you have calculated to the value found in the \(F\) table. Find the table value in the column corresponding to the degrees of freedom for \(MS_A\) termed \(\nu_1\), and the row corresponding to the degrees of freedom for \(MS_{S(A)}\) termed \(\nu_2\). If you want an \(\alpha\) level of .05, use the top number. If you want an \(\alpha\) level of .01, use the bottom number. If the calculated value of \(F\) is greater than the value found in the table, reject the null hypothesis, and accept the alternative hypothesis.

In our example the value found in the \(F\) table for \(df_A\) equal to 1 (column 1) and \(df_{S(A)}\) equal to 8 (row 8) is 5.32 for \(\alpha = .05\) and 11.26 for \(\alpha = .01\). Therefore at the .05 level our calculated \(F\) of 6.40 is larger than the table value and we can reject the null hypothesis that the diet makes no difference in the time it takes to run the maze. In other words, at the .05 level we can accept the alternative hypothesis and conclude that diet 2 improves speed on the maze. If, however, we had selected a significance level of .01, we could not reject the null hypothesis because 6.40 does not exceed 11.26. Therefore, at the .01 level we would have to suspend judgment.
5.2 Exercises

Exercise 5.1: Junk food consumption

A (hypothetical) study investigating eating patterns among students in different academic years was conducted the University of Texas at Dallas. The questionnaire, about daily junk food consumption, used a 5 point scale (5 – having snacks five or more times a day besides regular meals, 4 – four times, 3 – three times, 2 – twice, 1 – once or none). Six undergraduate and six graduate students completed this questionnaire. Their responses are listed in Table 5.1.

<table>
<thead>
<tr>
<th>Undergraduates</th>
<th>Graduates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

1. Indicate the following:

- **Independent variable and its levels:**
  \( A: \) ____________________________
  \( a_1: \) ____________________________
  \( a_2: \) ____________________________

- **Dependent variable:** \( Y \).

- **Null hypothesis, \( H_0 \).**

- **Alternative hypothesis, \( H_1 \).**
2. Compute the:
   - Degrees of freedom
     - Number of degrees of freedom total, $df_{total}$.
     - Number of degrees of freedom between groups, $df_{between} = df_A$.
     - Number of degrees of freedom within groups, $df_{within} = df_{S(A)}$.

3. Sums
   - Sum of all the scores, $Y_{..}$.
   - Sum of scores for $a_1$, $Y_{1.}$.
5.2 Exercises

- Sum of scores for $a_2, Y_2$.

4. Means

- grand mean, $M_.$.

- $a_1$ mean, $M_1$.

- $a_2$ mean, $M_2$.

5. Sum of squares

- Total sum of squares. $SS_{\text{total}}$. 
5.2 Exercises

- **Sum of squares between**, \( SS_{\text{between}} = SS_A \).

- **Sum of squares within**, \( SS_{\text{within}} = SS_{S(A)} \).

6. Mean squares

- **Mean square between**, \( MS_{\text{between}} = MS_A \).

- **Mean square within**, \( MS_{\text{within}} = MS_{S(A)} \).
7. The index $F$.

8. Fill in the ANOVA table.

<table>
<thead>
<tr>
<th>Source</th>
<th>$df$</th>
<th>$SS$</th>
<th>$MS$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between, $A$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within, $S(A)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. Find the critical value of $F$ at $\alpha=.05$ and $\alpha=.01$.

10. Are the results significant?

11. Write the conclusion using APA style.
Exercise 5.2: Journal of Easy Data and Nice Numbers

We have run an experiment with 4 groups and 5 subjects per group. The results are described in the table below.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>30</td>
<td>31</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>32</td>
<td>32</td>
<td>80</td>
</tr>
<tr>
<td>10</td>
<td>34</td>
<td>33</td>
<td>82</td>
</tr>
<tr>
<td>12</td>
<td>34</td>
<td>33</td>
<td>83</td>
</tr>
<tr>
<td>15</td>
<td>40</td>
<td>41</td>
<td>90</td>
</tr>
</tbody>
</table>

Compute the ANOVA table.
Exercise 5.3: Journal of Easy Data and Small Sized Groups

We have run an experiment with 3 groups and 2 subjects per group. The results are:

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>22</td>
</tr>
</tbody>
</table>

**Compute the ANOVA table.**
50  5.2 Exercises
6

ANOVA One factor, $S(A)$, Regression Approach

6.1 Summary

6.1.1 ANOVA and regression approach

An experimental design can be analyzed using an ANOVA, as has been previously shown. The analysis can also be accomplished by using regression. The main difference between ANOVA and regression is simply vocabulary and notation. One way to show the relationship between these two approaches is to use what is called the \textit{mean model}.

In the mean model we predict the performance of subjects from the mean of their group. In other words, the mean of a subject’s group becomes the independent variable $X$ (also called the predictor) and the actual subject score is the dependent variable $Y$ (what we want to predict). When a regression is done using these values it yields the following consequences:

- The mean of the predictor $M_X$, the mean of the dependent variable $M_Y$, and the grand mean of all the subject scores $M_\cdot$ will have the same value.

$$M_X = M_Y = M_\cdot$$

- The predicted value $\hat{Y}$ will always turn out to be the mean of the subject’s group also, $X = \hat{Y} = M_\cdot$.

- If the independent variable has an effect, the group mean should be a better predictor of subject behavior than the grand mean.

6.1.2 Example

1. The following steps can be used to demonstrate that using the group mean as a predictor $X$ will result in a predicted value of $\hat{Y}$ that is also equal to the group mean.

(a) Compute the mean of each group and the grand mean.
(b) Fill in the usual regression table shown below for all the scores. Use the group mean $M_a$ for $X$.

**TABLE 6.1** The following abbreviations are used: $x = X - M_X; y = Y - M_Y$.

<table>
<thead>
<tr>
<th></th>
<th>$X = M_a$</th>
<th>$x$</th>
<th>$x^2$</th>
<th>$y_{as}$</th>
<th>$y$</th>
<th>$y^2$</th>
<th>$x \times y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group N</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum$</td>
<td>$M_X$</td>
<td>$SS_X$</td>
<td>$M_Y$</td>
<td>$SS_Y$</td>
<td>$SCP_{XY}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Use the information from the table to compute the slope of the regression line. The slope will be equal to 1.

\[
b = \frac{SCP_{XY}}{SS_X} = 1 \quad \text{or alternatively} \quad b = \frac{\hat{\sigma}_Y}{\hat{\sigma}_X} r_{XY} = 1
\]

(d) Compute the intercept of the regression line. The intercept will be equal to 0.

\[
a = M_Y - bM_X = 0
\]

(e) Write the equation of the regression line.

\[
\hat{Y} = a + bX = 0 + 1 \times X = X = M_a.
\]

(f) This shows that the predicted value is also the group mean because $X$ is equal to $M_a$.

2. To determine the quality of the prediction compute the squared coefficient of correlation $r_{XY}^2$, between the predicted scores and the actual subject scores using the following steps.

(a) Compute the sum of squares total $SS_Y = \sum (Y - M_Y)^2$
6.2 Summary

(b) Compute the sum of squares regression

\[ SS_{\text{regression}} = \sum (\hat{Y} - M_Y)^2 \]

This result will have the same value as \( SS_{\text{between}} \) or \( SS_A \) when the problem is done using ANOVA.

(c) Compute the sum of squares residual

\[ SS_{\text{residual}} = \sum (Y - \hat{Y})^2 \]

This result will have the same value as \( SS_{\text{within}} \) or \( SS_{S(A)} \) when the problem is computed using ANOVA.

(d) Check that the sum of squares total is equal to the sum of squares regression plus the sum of squares residual.

(e) Compute \( r_{\hat{Y},Y}^2 \).

\[ r_{\hat{Y},Y}^2 = \frac{SS_{\text{regression}}}{SS_{\text{total}}} \]

3. Determine the degrees of freedom for regression and residual.

(a) \( df_{\text{regression}} \) will be the number of groups minus 1. The value will be the same as \( df_{\text{between}} = df_A \) in ANOVA.

(b) \( df_{\text{residual}} \) will be the number of total subjects minus the total number of groups. The value will be the same as \( df_{\text{within}} = df_{S(A)} \) in ANOVA.

4. Compute the index \( F \) for \( r_{\hat{Y},Y}^2 \).

\[ F = \frac{r_{\hat{Y},Y}^2}{1 - r_{\hat{Y},Y}^2} \times \frac{df_{\text{residual}}}{df_{\text{regression}}} \]

\[ = \frac{r_{\hat{Y},Y}^2}{1 - r_{\hat{Y},Y}^2} \times \frac{S - 2}{1} \]

where \( S \) is the total number of subjects.

(a) This \( F \) will be the same as the \( F \) computed using ANOVA.

(b) Locate the critical value for \( F \) in the \( F \) Table using \( df_{\text{regression}} \) for \( \nu_1 \) and \( df_{\text{residual}} \) for \( \nu_2 \).

(c) Compare the computed \( F \) to the table value as usual.
6.2 Exercises

Exercise 6.1: Predicting junk food consumption

This exercise shows that an experiment can be analyzed using either
the analysis of variance or regression approach. The result will be the
same. The following problem was previously presented as an exercise to
compute a one-factor analysis of variance. The problem is repeated here
for your convenience.

A (hypothetical) study has been done on the eating patterns among
students in different academic years at the University of Texas at Dallas.
The questionnaire, about daily junk food consumption, used a 5 point
scale (5 – having snacks five or more times a day besides regular meals, 4 –
four times, 3 – three times, 2 – twice, 1 – once or none). Six undergraduate
and six graduate students completed this questionnaire. Their responses
were:

<table>
<thead>
<tr>
<th>Undergraduates</th>
<th>Graduates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

For this exercise use the regression approach.

1. Demonstrate that, for the data above, the best prediction of an individ-
ual’s score is the mean of that individual’s group.

   (a) Compute the mean of each group and the grand mean.
(b) Fill in Table 6.2 using the group mean $M_a$ for $X$.

**TABLE 6.2** The following abbreviations are used: $x = X - M_X; y = Y - M_Y$.

<table>
<thead>
<tr>
<th></th>
<th>$X = M_a$</th>
<th>$x$</th>
<th>$x^2$</th>
<th>$y_{as}$</th>
<th>$y$</th>
<th>$y^2$</th>
<th>$x \times y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undergraduate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graduate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sum$</th>
<th>$M_X$</th>
<th>$SS_X$</th>
<th>$M_Y$</th>
<th>$SS_Y$</th>
<th>$SCP_{XY}$</th>
</tr>
</thead>
</table>

(c) Use the information from the table to compute the slope of the regression line.

(d) Compute the intercept of the regression line.

(e) Write the equation of the regression line.
2. Compute the squared coefficient of correlation $r_{Y,Y}^2$ between the predicted scores and the actual subject scores using the following steps.

(a) Compute the sum of squares total.

(b) Compute the sum of squares regression.

(c) Compute the sum of squares residual.

(d) Check that the sum of squares total is equal to the sum of squares regression plus the sum of squares residual.

(e) Compute $r_{Y,Y}^2$. 
3. **What are the values for** $df_{\text{regression}}$ **and** $df_{\text{residual}}$?

$$df_{\text{regression}} = \text{______________________________}$$

$$df_{\text{residual}} = \text{______________________________}$$

4. **Compute the index** $F$ **for** $r^2_{Y,Y'}$. 

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6.2 Exercises
7

Contrasts and Comparisons - Summary

7.1 Summary

7.1.1 Comparisons

The $F$ ratio provides an indication of the global effect of the independent variable on the dependent variable. It can show that there is a reliable influence of the independent variable on the dependent variable. Comparisons are used to analyze this effect in more detail by comparing the means of different experimental groups. There are two types of comparisons:

1. *A priori comparisons*: These comparisons are planned before running the experiment. They are chosen to see if your results support or do not support theories. They are usually few in number. They may be orthogonal (independent) or non-orthogonal.

2. *A posteriori comparisons*: These comparisons are not pre-planned, but decided on after the experiment has been run. They are done to see if some of the result patterns that were not anticipated are reliable.

7.1.2 Contrasts

A *contrast* is a type of comparison with one degree of freedom. All contrasts are comparisons, but not all comparisons are contrasts. The other type of comparison is called a *sub-design*. The number of possible independent (orthogonal) contrasts is the same as the number of degrees of freedom of the sum of squares for the independent variable $A$, $df_A = A - 1$. Each contrast is denoted by the Greek letter psi ($\psi$) with a subscript. Therefore, the first contrast is denoted $\psi_1$, the second $\psi_2$, and so on.

7.1.3 Contrast Coefficients

When a contrast is performed, the means of all the different experimental groups are weighted by coefficients. These contrast coefficients are de-
noted $C_a$ where $a$ indicates the experimental group. Therefore, the coefficient of the first group is $C_1$, the coefficient of the second group is $C_2$, and so on. The sum of all of these coefficients is 0, formally:

$$\sum_{a=1}^{A} C_a = 0.$$ 

Furthermore, a second subscript can be placed on the $C$ to identify the contrast. If coefficients are constructed for contrast 1 ($\psi_1$) the second subscript on each coefficient is 1. Therefore the coefficient $C_{1,1}$ denotes the coefficient for the first group in the first contrast, and $C_{2,1}$ denotes the coefficient for the second group in the first contrast. To denote the third group in the fourth contrast we would write $C_{3,4}$.

### 7.1.4 Determining Contrast Coefficients

The objective in determining the contrast coefficients is to find a set of integers (whole numbers, positive or negative), one integer for each experimental group such that:

- The sum of these numbers is equal to 0.
- The numbers will weight the means of each group appropriately for the hypothesis being tested.

Suppose that you are interested in studying the effect of sound environment on the productivity of workers in a factory. You design an experiment to test the effects of different musical backgrounds on the number of widgets produced at the end of a week. There are three groups in the experimental design:

1. A control group for which no music is played.
2. A group for which some happy music is played.
3. A group for which some sad music is played.

If your theory is that the presence of music (no matter if it is happy or sad) will affect the productivity, you will pre-plan contrast 1 ($\psi_1$) to compare the means of the control and music groups. The contrast coefficients are:

$$C_{1,1} = 2, \quad C_{2,1} = -1, \quad C_{3,1} = -1.$$ 

The null hypothesis is that there is no difference between the means that we are contrasting. It is symbolically represented as:

$$H_0 : \quad \psi_1 = 0 \quad \text{where} \quad \psi_1 = 2\mu_1 - \mu_2 - \mu_3$$

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If another theory is that listening to happy music will produce different results than listening to sad music, contrast 2 ($\psi_2$) will be planned to compare the means of happy and sad music groups. The contrast coefficients in this contrast are:

$$C_{1,2} = 0, \quad C_{2,2} = 1, \quad C_{3,2} = -1.$$  

### 7.1.5 Orthogonal Contrasts

When a family of contrasts is orthogonal, each contrast in the family is independent of all the other contrasts in the family. In other words, any given contrast is not correlated to any other one: its correlation coefficient with each of the other contrasts is equal to zero ($r = 0$). To check the orthogonality of two contrasts, compute the cross product. If the cross product is equal to zero the two contrasts are orthogonal. To compute the cross product of two contrasts (say $\psi_1$ and $\psi_2$), multiply the coefficients in corresponding positions and then sum all the products. Multiply the first coefficient of the first contrast by the first coefficient of the second contrast. Do the same for the second, third, fourth, etc. coefficients of the two contrasts. When this has been done for all coefficients take the sum. Formally this is expressed as

$$\sum_{a=1}^{A} C_{a,1}C_{a,2}$$

where $a$ is an index denoting the experimental group.

To test for orthogonality the two contrasts in the example above, compute the cross product as follows:

$$(2 \times 0) + (-1 \times 1) + (-1 \times -1) = 0 - 1 + 1 = 0$$

Since the cross product is equal to 0, the contrasts are orthogonal.

If there are more than two contrasts in a family, each possible pair must be checked for orthogonality in this manner. Remember that no more than $A - 1$ contrasts can be orthogonal to each other for an independent variable with $A$ levels.

### 7.1.6 Converting a Research Hypothesis into Contrast Coefficients

Imagine that you want to test the following hypothesis: Subjects in group 2 (happy music) should produce twice as many widgets as subjects in group 1 (no music) and subjects in group 3 (sad music) should produce half of what subjects in group 1 produce. To write the contrast coefficients, follow the steps below:

- **Step 1:** Rank the groups to match the prediction for the means.
7.2 Exercises

\[
\begin{array}{ccc}
C_1 & C_2 & C_3 \\
2 & 4 & 1
\end{array}
\]

- **Step 2:** Compute the mean of the ranks.

\[ M = \frac{7}{3} \]

- **Step 3:** Subtract this mean from each rank.

\[
\begin{align*}
\frac{6}{3} - \frac{7}{3} & \quad \frac{12}{3} - \frac{7}{3} & \quad \frac{3}{3} - \frac{7}{3}
\end{align*}
\]

which gives:

\[
\begin{align*}
-\frac{1}{3} & \quad \frac{5}{3} & \quad -\frac{4}{3}
\end{align*}
\]

*Note:* To add or subtract two fractions, the two fractions need to have the same denominator. For example, to subtract \(\frac{7}{3}\) from 2, you need first to multiply 2 by \(\frac{3}{3}\) which will give you \(\frac{6}{3}\) and now you can subtract the numerator of \(\frac{7}{3}\) from the numerator of \(\frac{6}{3}\) which gives \(-\frac{1}{3}\).

- **Step 4:** Transform the fractions to integers

\[
\begin{align*}
-\frac{1}{3} \times 3 & \quad \frac{5}{3} \times 3 & \quad -\frac{4}{3} \times 3
\end{align*}
\]

which gives

\[
\begin{align*}
-1 & \quad 5 & \quad -4
\end{align*}
\]

which finally gives us the contrast coefficients that we were looking for.

### 7.2 Exercises

**Exercise 7.1:** Contrast coefficients

An experimenter wants to test the following predictions:
Write down the $C_a$ values for the contrasts that the experimenter should use.
Exercise 7.2: Checking for independence

Check the independence of these contrasts. Two contrasts are independent, or orthogonal if their cross product is equal to zero.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1</th>
<th>-4</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Exercise 7.3: Multiplication tables

A total of 15 third grade subjects are randomly assigned to one of 3 groups for an introduction to the multiplication tables. Each group is instructed using a different technique as follows:

- Group 1—Rote memorization and drill.
- Group 2—Manipulation of wooden blocks and counting.
- Group 3—Repeated addition (e.g., $3 \times 8 = 8 + 8 + 8$).

The experimenter wants to test the hypothesis that using the wooden blocks will be the most beneficial approach, and that rote memorization is the poorest approach. Specifically, he theorizes that the group using the wooden blocks (Group 2) will do four times as well on a test as the group using rote memorization and drill (Group 1) and that Group 3 will do twice as well as Group 1.

1. **Sketch the shape of the experimenter’s prediction.**

2. **Convert the shape to ranks.**
3. Convert the ranks to contrast coefficients.
8

ANOVA, Contrasts, and Comparisons - Computation

8.1 Summary

8.1.1 Computing $SS_{\psi}$, $MS_{\psi}$, and $F_{\psi}$

The sum of squares for a contrast is denoted $SS_{\psi}$. It is computed as:

$$SS_{\psi} = \frac{S(\sum C_{a}M_{a.})^{2}}{\sum C_{a}^{2}} = \frac{S\psi^{2}}{\sum C_{a}^{2}}.$$

The number of degrees of freedom for a contrast, denoted $df_{\psi}$, is always equal to 1.

$$df_{\psi} = 1.$$

The mean square for a contrast is denoted $MS_{\psi}$ and is equal to the sum of squares for the contrast since there is only one degree of freedom.

$$MS_{\psi} = \frac{SS_{\psi}}{df_{\psi}} = \frac{SS_{\psi}}{1} = SS_{\psi}.$$

The null hypothesis for a contrast is tested by computing the criterion $F_{\psi}$ as the ratio of the $MS_{\psi}$ to the measure of error of the analysis of variance, $MS_{S(A)}$

$$F_{\psi} = \frac{MS_{\psi}}{MS_{S(A)}},$$

which is distributed, under the null hypothesis as a Fisher distribution with 1 and $A(S - 1)$ degrees of freedom. If the probability associated with $F_{\psi}$ is smaller than the alpha level or if $F_{\psi}$ is greater than the critical $F$ [i.e., the $F$ value found in the $F$ table for $\nu_1 = 1$ and $\nu_2 = A(S - 1)$], then the null hypothesis is rejected.

8.1.2 Example

Let us go back to the example that investigated the effect of sound environment on the productivity of factory workers. You obtained the following results from three groups of subjects with 3 subjects in each group (see Table 8.1).
You want to test the first hypothesis—the presence of music will affect productivity—which can be translated into the following contrast coefficients:

\[ C_{1,1} = 2, \quad C_{2,1} = -1, \quad C_{3,1} = -1. \]

First you need to compute the mean of each group:

\[ M_1 = 3, \quad M_2 = 7, \quad M_3 = 2 \]

The steps for the computation of \( SS_\psi \) are given in the following table:

<table>
<thead>
<tr>
<th>Group</th>
<th>( M_a )</th>
<th>( C_a )</th>
<th>( C_a M_a )</th>
<th>( C_a^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>-1</td>
<td>-7</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\sum \quad 0 \quad -3 \quad 6
\]

From that table, \( SS_\psi \) is

\[
SS_\psi = \frac{S(\sum C_a M_a.)^2}{\sum C_a^2} = \frac{3 \times (-3)^2}{6} = \frac{3 \times 9}{6} = 4.50
\]

and therefore:

\[
MS_\psi = \frac{SS_\psi}{df_\psi} = \frac{4.5}{1} = 4.50
\]

Before computing the \( F \) ratio, we need to compute the mean square of error, \( MS_{S(A)} \). Recall that

\[
SS_{S(A)} = \sum (Y_{sa} - M_a.)^2
\]

\[
= (1 - 3)^2 + (5 - 3)^2 + (3 - 3)^2
\]

\[
+ (8 - 7)^2 + (6 - 7)^2 + (7 - 7)^2
\]

\[
+ (2 - 2)^2 + (3 - 2)^2 + (1 - 2)^2
\]

\[
= (-2)^2 + (2)^2 + (0)^2 + (1)^2 + (-1)^2 + (0)^2 + (0)^2 + (1)^2 + (-1)^2
\]

\[
= 4 + 4 + 0 + 1 + 1 + 0 + 0 + 1 + 1 = 10
\]

\[
SS_{S(A)} = 10
\]

\[
MS_{S(A)} = \frac{SS_{S(A)}}{df_{S(A)}} = \frac{10}{5} = 2.00
\]
\begin{align*}
  &= 4 + 4 + 0 + 1 + 1 + 0 + 0 + 1 + 1 \\
  &= 12.00
\end{align*}

and that

\[ MS_{S(A)} = \frac{SS_{S(A)}}{df_{S(A)}} = \frac{12}{3(3 - 1)} = \frac{12}{6} = 2.00 \]

\[ F_\psi \text{ is then given by:} \]

\[ F_\psi = \frac{MS_\psi}{MS_{S(A)}} = \frac{4.5}{2} = 2.25 \]

and follows a Fisher distribution with \( \nu_1 = 1 \) and \( \nu_2 = 6 \) degrees of freedom. The critical value found in the table is equal to 5.99, for \( \alpha = .05 \). Since \( F_\psi \) is smaller than the critical value of \( F \) we cannot reject the null hypothesis and hence we suspend judgment.

### 8.2 Exercises

**Exercise 8.1:** \( 7 \times 8 \neq 58! \)

A total of 15 third grade subjects are randomly assigned to one of 3 groups for an introduction to the multiplication tables. Each group is instructed using a different technique as follows:

- Group 1— Rote memorization and drill.
- Group 2— Manipulation of wooden blocks and counting.
- Group 3— Repeated addition (e.g., \( 3 \times 8 = 8 + 8 + 8 \)).

The experimenter wants to test the hypothesis that using the wooden blocks will be the most beneficial approach, and that rote memorization is the poorest approach. Specifically, he theorizes that the group using the wooden blocks (Group 2) will do four times as well on a test as the group using rote memorization and drill (Group 1) and that Group 3 will do twice as well as Group 1.

The experiment is conducted for 3 weeks, and then a test of multiplication number facts is administered to all the subjects. The results of the test are given in Table 8.2. The score is the number of problems correct out of 10.

1. **What are the contrast coefficients?**
2. Determine the mean for each group.
3. Compute $F_{\psi_1}$ (the $MS_{S(A)}$ is 2).
4. Is support found for the experimenter’s prediction?

Exercise 8.2: Smart cookies

Gâteaux analyzes the data from her latest experiment with an $S(A)$ design. Factor $A$ is the type of cookies eaten by the subjects during their midterm exam. Upon arrival in the classroom, subjects were randomly assigned to a “cookie” condition. A box of cookies were placed on each desk and the students were told that they could eat as many cookies as they wanted during the exam. The exam score for each student is then recorded. Note, the maximum number of possible points for this exam is 10.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oatmeal</td>
<td>Oreos</td>
<td>Choc. Chip</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
</tbody>
</table>

1. Complete the ANOVA table.
2. **State the conclusion in APA style.**

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

3. **Why can the experimenter reach this conclusion?**

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

4. **What is the proportion of variance explained by the independent variable?**

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

5. **Is this value of $R^2_{Y,A}$ statistically reliable?**

   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
6. Further, analyze the results from the “cookie” experiment using the regression approach. First, what is the equation for the prediction?
8.2 Exercises

7. How reliable is this prediction?

8. If the experimenter had added a fourth group with an experimental group mean of 6, what would be the predicted score for this group? Why?
Prior to conducting the “cookie” experiment, Gâteaux felt she had reasons to doubt the effectiveness of oatmeal cookies, while suspecting that the cocoa bean may act as a performance enhancer. She runs her contrast analyses with the following coefficients. Remember, Group 1: Oatmeal cookies; Group 2: Oreos; and Group 3: Chocolate chip cookies.

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td>-2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

9. Are these contrasts orthogonal?
10. What can the experimenter conclude?
11. Further, run the contrast analyses using the multiple regression approach. What conclusions can be drawn from these analyses? (Use the contrast coefficients given in Table 8.3.

<table>
<thead>
<tr>
<th>Table 8.3 Contrast coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
</tr>
<tr>
<td>$\psi_1$</td>
</tr>
<tr>
<td>$\psi_2$</td>
</tr>
</tbody>
</table>
Suppose that, instead of the previous contrast analyses, Gâteaux predicted that indeed Oreo cookie consumption would result in higher test scores. But she also wanted to put to rest a longstanding argument between oatmeal and chocolate chip aficionados. Which type of cookie would stack up against the superiority of the Oreo cookie? Choosing $\alpha = .01$, she further analyzed the data with the following contrast coefficients:

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td>$-1$</td>
<td>$1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>$0$</td>
<td>$1$</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

12. Are these contrasts orthogonal?
13. What can the experimenter conclude?
14. Further, run the contrast analyses using the multiple regression approach. What conclusions can be drawn?

\[
\begin{array}{ccc}
C_1 & C_2 & C_3 \\
\psi_1 & -1 & 1 & 0 \\
\psi_2 & 0 & 1 & -1 \\
\end{array}
\]

**TABLE 8.4** Value for Contrast Analysis, Multiple Regression Approach.

<table>
<thead>
<tr>
<th>Iden</th>
<th>Y</th>
<th>y</th>
<th>y^2</th>
<th>C_{a,1}</th>
<th>C_{a,1}^2</th>
<th>yC_{a,1}</th>
<th>C_{a,1}</th>
<th>C_{a,2}^2</th>
<th>yC_{a,2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-3</td>
<td>9</td>
<td>-1</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-2</td>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-2</td>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>\sum</td>
<td>0</td>
<td>0</td>
<td>42</td>
<td>0</td>
<td>8</td>
<td>16</td>
<td>0</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

**SS_Y** | **SS_{C_{a,1}}** | **SCP_{YC_{a,1}}** | **SS_{C_{a,2}}** | **SCP_{yC_{a,2}}**
Exercise 8.3: “Mean” ANOVA

The following data comes from an experiment reported in the *Journal of Generic and Fictional Psychology*. This journal (which, as the name indicates, is a completely fictitious APA journal) specializing in reporting data without any explanation. In a fictitious, but thrilling paper, Toto (2002) reports the following data:

- Experimental design: $S(A)$.
- $A = 5$.
- $S = 9$.
- $MS_{S(A)} = 15.00$

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

1. Use the above information to build back the ANOVA table.
2. After having run the ANOVA and looked at the results, the experimenter saw some unexpected results and decided to run some further contrast analyses. The contrast coefficients for one of the contrasts are given in the table below. What approach do you think the experimenter should use to evaluate this contrast? Does this contrast fit the results more than chance would?

<table>
<thead>
<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_1$</td>
<td>-2</td>
<td>-2</td>
<td>-2</td>
<td>+3</td>
<td>+3</td>
</tr>
</tbody>
</table>
3. In addition, the experimenter wanted to look at the pairwise differences of the means. She decided to use both the Tukey and Newman-Keuls test. Could you do the same (and do it!)?
4. Find two \textit{different} sets of four orthogonal contrasts for this design.
5. Further, show *numerically* for this example that when \( C_a = (M_a - M_.) \) then \( SS_A = SS_{\text{comp.}} \).
C'est tout!

Exercise 8.4: Once upon a time in Aschaffenburger!

This story is taken from Spitzer (1999, p.220). Aschaffenburg (1899, no typo here, the work is more than a century old), was a German (as the same suggests¹) psychologist. To explore the effect of fatigue on cognition, he used the technique of free association where subjects associate an answer word to a stimulus word. He categorized an association as conceptual when the answer and the stimulus are related by their meaning (e.g., answer variance for the stimulus mean); he categorized an association as “pun-ish”² (or following Spitzer’s terminology as clang) when the answer and the stimulus are related by their sound (e.g., answer queen for stimulus mean). His idea was that, when subjects are tired, their word associations should be more pun-ish (or less conceptual). In order to explore this hypothesis, Aschaffenburg used subjects working on night shifts. Subjects gave word associations to lists made of one hundred words at four different times of the night: 9PM, midnight, 3AM and 6AM. His dependent variable was the number of pun-ish associations.

He obtains the data given in Table 8.5.

### Table 8.5 Nice Numbers: data inspired by Spitzer (1999, p. 220, originally from Aschaffenburg, 1899).

<table>
<thead>
<tr>
<th></th>
<th>9PM</th>
<th>Midnight</th>
<th>3AM</th>
<th>6AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>22</td>
<td>24</td>
<td>34</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>22</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>10</td>
<td>17</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>9</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>12</td>
<td>13</td>
<td>19</td>
</tr>
</tbody>
</table>

| Ya    | 45  | 75      | 85  | 135 |

¹I am not kidding this is a real name!
²Isn't it funny?
1. Plot the results of this experiment.

2. Is there an effect of the time of the night on the number of pun-ish associations?
3. What are the pairs of conditions that show a reliable difference?
4. When looking at the results, I have the impression that the average number of pun-ish associations increases as a linear function of the time of the night. What contrast coefficient should I use to test that idea?
5. How well can we predict the group result with this linear hypothesis?

6. How well can we predict the individual results with this linear hypothesis?

7. Are these predictions reliable?
8. Create an example with the same error mean square, but no effect of $A$ at all.
8.2 Exercises
9. Create an example with the same error mean square and a small effect of $A$ which is *not* statistically reliable.
10. Create an example with an error mean square four times as large as the one observed (Hint: $4 = 2^2$), and the same effect of $A$ as in the original data.
8.2 Exercises
9

ANOVA Two-Factors $S(A \times B)$: Main Effects and Interaction

9.1 Summary

9.1.1 Plotting the Means

In a two-factor between-subjects experimental design [$S(A \times B)$], we manipulate two independent variables. If there are $A$ levels of one independent variable and $B$ levels of the other independent variable then there are $A \times B$ experimental conditions or experimental groups. For example, with $A = 2$ and $B = 2$ there are $2 \times 2 = 4$ conditions. This particular design is often called a $2 \times 2$ factorial or crossed design.

To analyze the results of a factorial design, we compare the group means from the different combinations of these conditions. The following examples illustrate the different types of outcomes can result in a $2 \times 2$ factorial design. In every example, there were four subjects assigned to each of the four conditions (therefore $S = 4$). The grand mean of the dependent variable is 10 for all the examples. Each example gives a table containing the mean for each condition, the mean for each level of each independent variable (the so-called marginal means), and the grand mean. The group means are also graphically displayed.

Remember, an effect of an independent variable will show up as a difference in the marginal means. Therefore, the column means will be different if there is a main effect of independent variable $A$, and will be the same if there is no main effect of independent variable $A$. Similarly, the row means will be different if there is a main effect of the independent variable $B$, and will be the same if there is no main effect of independent variable $B$. These examples represent an ideal situation. Of course, in “real life” differences that are not statistically significant may occur when there is no effect of the independent variable. An interaction is defined as existing when the effects of one independent variable are not the same over the different levels of the other independent variable.

The means of the different experimental conditions are traditionally plotted in the following manner:
1. Hashmarks are placed on the vertical, or \(Y\), axis to indicate the values of the dependent variable.

2. Hashmarks are placed on the horizontal, or \(X\), axis to indicate the levels of one of the independent variables.

3. The means of one of the levels of the second independent variable are plotted using a dot over the hashmarks of the first independent variable on the \(X\) axis. These dots are then joined with a straight line and labeled.

4. The means of the other levels of the second independent variable are also plotted using dots over the hashmarks of the first independent variable on the \(X\) axis and joined with lines and labeled.

### 9.2 No experimental effect

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>b2</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Means</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
9.3 Main effect of independent variable $B$

\begin{center}
\begin{tabular}{lcccc}
 & a1 & a2 & Means \\
 b1 & 8 & 8 & 8 \\
b2 & 12 & 12 & 12 \\
\hline
Means & 10 & 10 & 10 \\
\end{tabular}
\end{center}

9.4 Main effect of independent variable $A$

\begin{center}
\begin{tabular}{lcccc}
 & a1 & a2 & Means \\
 b1 & 8 & 12 & 10 \\
b2 & 8 & 12 & 10 \\
\hline
Means & 8 & 12 & 10 \\
\end{tabular}
\end{center}

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9.5 Main effect of both independent variables $A$ AND $B$

![Graph showing main effect of both independent variables]

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>12</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>8</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>

9.6 Interaction with no main effects

![Graph showing interaction with no main effects]

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>8</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>b2</td>
<td>12</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Means</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
9.7 Interaction with main effect of independent variable \( B \)

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>16</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>b2</td>
<td>4</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Means</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

9.8 Interaction with main effect of independent variable \( A \)

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>b2</td>
<td>6</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>Means</td>
<td>8</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>

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9.9 Interaction with main effects of $A$ and $B$

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>b2</td>
<td>2</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Means</td>
<td>8</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>
Exercise 9.1: Means and outcome

For each of the following tables of means, graph the means for each condition and determine the type of outcome.

1. Outcome is: ________________________________________________________________

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>5</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>15</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Means

2. Outcome is: ________________________________________________________________
### 9.9 Interaction with main effects of $A$ and $B$

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>20</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

**Means**

3. **Outcome is:**

\[
\begin{array}{|c|c|}
\hline
\text{a1} & \text{a2} \\
\hline
10 & 20 \\
5 & 15 \\
\hline
\end{array}
\]

**Means**

4. **Outcome is:**

\[
\begin{array}{|c|c|}
\hline
\text{a1} & \text{a2} \\
\hline
10 & 20 \\
5 & 15 \\
\hline
\end{array}
\]

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9.9 Interaction with main effects of $A$ and $B$

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>8</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Means</td>
</tr>
</tbody>
</table>

5. Outcome is: ________________________________

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>10</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Means</td>
</tr>
</tbody>
</table>

6. Outcome is: ________________________________

© 2010 Abdi, Williams, Edelman, Valentin, & Posamentier
### Table 9.9: Interaction with main effects of A and B

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Means</td>
</tr>
</tbody>
</table>

7. Outcome is: ________

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>10</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Means</td>
</tr>
</tbody>
</table>

8. Outcome is: ________
<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Exercise 9.2: Creating means

In a $2 \times 2$ factorial design $S(A \times B)$ the grand mean of the dependent variable is 12. Create means for each condition when the outcome is:

1. No effects.

<table>
<thead>
<tr>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

2. Main effect of $A$.

<table>
<thead>
<tr>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

3. Main effect of $B$.

<table>
<thead>
<tr>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

4. Main effect of $A$ and $B$.

<table>
<thead>
<tr>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Means</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

5. Interaction with no main effects.
9.9 Interaction with main effects of $A$ and $B$

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Means 12

6. Interaction with main effect of $B$.

<table>
<thead>
<tr>
<th></th>
<th>a1</th>
<th>a2</th>
<th>Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>b1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Means 12
9.9 Interaction with main effects of $A$ and $B$
10

ANOVA Two-Factors, $S(A \times B)$, Computation

10.1 Summary

A researcher was interested in the difference between children and adults’ motor skills, as well as looking at the possible existence of a gender difference. She got together a group of 4th graders and a group of Ph.D. students to test their video game skills when playing a brand new game. This video game requires the integrative and cooperative movements of eyes, hands, and fingers. The motor competence was measured by the highest level of difficulty of the game that a subject mastered. The highest level of difficulty is 5 and the lowest 1. The subjects, half males and half females, played the game for one hour. The following scores were recorded:

<table>
<thead>
<tr>
<th>TABLE 10.1 Video games scores for 4th graders and Ph.D. students.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th grade</td>
</tr>
<tr>
<td>Male</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

10.1.1 Notation Review

- In this experiment, there are two factors (i.e., two independent variables): gender and age. We will call these factors respectively $A$, with a number of levels $A$, and $B$, with a number of levels $B$. Here we have two levels for both $A$ and $B$ denoted respectively $a_1$ for male, $a_2$ for female and $b_1$ for 4th graders, $b_2$ for Ph.D students. Consequently the design is a $2 \times 2$ between subjects design.
- There are 3 subjects per group, $S = 3$. A group is defined as one $ab$
10.1 Summary

There are $A \times B = 2 \times 2 = 4$ groups denoted $a_1b_1$, $a_1b_2$, $a_2b_1$, $a_2b_2$. The score for a particular subject $s$ in the group $ab$ is denoted $Y_{abs}$.

- The dependent variable $Y$ is the highest level of difficulty mastered by a subject.

- Since we have two independent variables, there are three null hypotheses:
  
  - for $A$: gender has no effect on the highest level of difficulty mastered,
  
  - for $B$: age has no effect on the highest level of difficulty mastered,
  
  - for $AB$: there is no interaction between the two independent variables.

- Therefore, there are three alternative hypotheses:
  
  - for $A$: gender affects the highest level of difficulty mastered,
  
  - for $B$: age affects the highest level of difficulty mastered,
  
  - for $AB$: there is an interaction between the two independent variables.

10.1.2 Degrees of freedom

- The total number of degrees of freedom is 1 less than the product of $A$ (the number of levels of $A$), $B$ (the number of levels of $B$) and $S$ (the number of subjects per group):
  
  $$ df_{total} = (A \times B \times S) - 1 = (2 \times 2 \times 3) - 1 = 11 $$

- The number of degrees of freedom for the main effect of $A$ is 1 less than $A$, the number of levels of $A$:
  
  $$ df_A = A - 1 = 2 - 1 = 1 $$

- The number of degrees of freedom for the main effect of $B$ is 1 less than $B$, the number of levels of $B$:
  
  $$ df_B = B - 1 = 2 - 1 = 1 $$
• The number of degrees of freedom for the interaction between $A$ and $B$ is the product of 1 less than $A$, the number of levels of $A$, and 1 less than $B$, the number of levels of $B$:

$$df_{AB} = (A - 1) \times (B - 1) = (2 - 1) \times (2 - 1) = 1$$

• The number of degrees of freedom within groups is the product of $A$, the number of levels of $A$, $B$, the number of levels of $B$ and 1 less than $S$, the number of subjects per group:

$$df_{S(AB)} = A \times B \times (S - 1) = 2 \times 2 \times (3 - 1) = 8$$

10.1.3 Sums

• Sum of all the scores:

$$Y_{..} = \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{s=1}^{S} Y_{abs}$$

$$= 4 + 4 + 4 + 1 + 3 + 2 + 5 + 4 + 3 + 2 + 2 + 2$$

$$= 36.00$$

• Sum of scores for the experimental group $ab$:

$$Y_{ab.} = \sum_{s=1}^{S} Y_{abs}$$

Since both $A$ and $B$ have two levels, the previous formula defines actually a set of $2 \times 2 = 4$ $Y_{ab.}$ terms:

$$Y_{11.} = \sum_{s=1}^{S} Y_{11s} = 4 + 4 + 4 = 12.00$$

$$Y_{12.} = \sum_{s=1}^{S} Y_{12s} = 1 + 3 + 2 = 6.00$$

$$Y_{21.} = \sum_{s=1}^{S} Y_{21s} = 5 + 4 + 3 = 12.00$$

$$Y_{22.} = \sum_{s=1}^{S} Y_{22s} = 2 + 2 + 2 = 6.00$$
• Sum of scores in condition \( a \):

\[
Y_{a..} = \sum_{b=1}^{B} \sum_{s=1}^{S} Y_{abs}
\]

Since \( A \) has two levels, the previous formula defines actually a set of \( 2 Y_{a..} \) terms:

\[
Y_{1..} = \sum_{b=1}^{B} \sum_{s=1}^{S} Y_{1bs} = 4 + 4 + 4 + 1 + 3 + 2 = 18.00
\]

\[
Y_{2..} = \sum_{b=1}^{B} \sum_{s=1}^{S} Y_{2bs} = 5 + 4 + 3 + 2 + 2 + 2 = 18.00
\]

• Sum of scores in condition \( b \):

\[
Y_{.b.} = \sum_{a=1}^{A} \sum_{s=1}^{S} Y_{abs}
\]

Since \( B \) has two levels, the previous formula defines actually a set of \( 2 Y_{.b.} \) terms:

\[
Y_{1.} = \sum_{a=1}^{A} \sum_{s=1}^{S} Y_{1as} = 4 + 4 + 4 + 5 + 4 + 3 = 24.00
\]

\[
Y_{2.} = \sum_{a=1}^{A} \sum_{s=1}^{S} Y_{2as} = 1 + 3 + 2 + 2 + 2 + 2 = 12.00
\]

10.1.4 Means

• Grand mean: The sum of all the scores divided by the total number of subjects.

\[
M_{..} = \frac{Y_{..}}{A \times B \times S} = \frac{36.00}{2 \times 2 \times 3} = 3.00
\]

• Mean for the experimental group \( ab \):

\[
M_{ab.} = \frac{1}{S} \times Y_{ab}
\]

Since both \( A \) and \( B \) have two levels, the previous formula defines actually a set of \( 2 \times 2 = 4 M_{ab.} \) terms:
\[ M_{11.} = \frac{1}{S} \times Y_{11.} = \frac{12.00}{3} = 4.00 \]
\[ M_{12.} = \frac{1}{S} \times Y_{12.} = \frac{6.00}{3} = 2.00 \]
\[ M_{21.} = \frac{1}{S} \times Y_{21.} = \frac{12.00}{3} = 4.00 \]
\[ M_{22.} = \frac{1}{S} \times Y_{22.} = \frac{6.00}{3} = 2.00 \]

- Mean for condition \( a \):

\[ M_{a..} = \frac{1}{B \times S} \times Y_{a..} \]

Since \( A \) has two levels, the previous formula defines actually a set of 2 \( M_{a..} \) terms:

\[ M_{1..} = \frac{1}{B \times S} \times Y_{1..} = \frac{18.00}{2 \times 3} = 3.00 \]
\[ M_{2..} = \frac{1}{B \times S} \times Y_{2..} = \frac{18.00}{2 \times 3} = 3.00 \]

- Mean for condition \( b \):

\[ M_{b..} = \frac{1}{A \times S} \times Y_{b..} \]

Since \( B \) has two levels, the previous formula defines actually a set of 2 \( M_{b..} \) terms:

\[ M_{1..} = \frac{1}{A \times S} \times Y_{1..} = \frac{24.00}{2 \times 3} = 4.00 \]
\[ M_{2..} = \frac{1}{A \times S} \times Y_{2..} = \frac{12.00}{2 \times 3} = 2.00 \]

- Table of means:

<table>
<thead>
<tr>
<th></th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>means</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_1 )</td>
<td>( M_{11.} )</td>
<td>( M_{21.} )</td>
<td>( M_{1..} )</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>( M_{12.} )</td>
<td>( M_{22.} )</td>
<td>( M_{2..} )</td>
</tr>
<tr>
<td>means</td>
<td>( M_{1..} )</td>
<td>( M_{2..} )</td>
<td>( M_{..} )</td>
</tr>
</tbody>
</table>
For the experiment, the mean table is as follows:

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>means</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$b_2$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>means</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Note: Actually \textit{when the design is balanced} (recall that a design is balanced when there is the same number of observations \textit{per} experimental condition), this table is a good way to check your results. Indeed the last number in each row (and respectively in each column) is the mean of the previous numbers in the row (respectively in the column).

\subsection*{10.1.5 Sum of squares}

- \textit{Total} sum of squares:

\[
SS_{\text{total}} = \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{s=1}^{S} (Y_{abs} - M_{..})^2
\]

\[
= (4 - 3)^2 + (4 - 3)^2 + (4 - 3)^2 + (1 - 3)^2 + (3 - 3)^2 + (2 - 3)^2 + \\
(5 - 3)^2 + (4 - 3)^2 + (3 - 3)^2 + (2 - 3)^2 + (2 - 3)^2 + (2 - 3)^2
\]

\[
= 1 + 1 + 1 + 4 + 0 + 1 + 4 + 1 + 0 + 1 + 1 + 1
\]

\[
= 16.00
\]

- Sum of squares for the \textit{main effect} of $A$:

\[
SS_A = B \times S \times \sum_{a=1}^{A} (M_{a..} - M_{..})^2
\]

\[
= 2 \times 3 \times [(3 - 3)^2 + (3 - 3)^2]
\]

\[
= 6 \times (0 + 0)
\]

\[
= 0.00
\]

- Sum of squares for the \textit{main effect} of $B$:

\[
SS_B = A \times S \times \sum_{b=1}^{B} (M_{b..} - M_{..})^2
\]

\[
= 2 \times 3 \times [(4 - 3)^2 + (2 - 3)^2]
\]
= 6 \times (1 + 1) \\
= 12.00 \\

- Sum of squares for the interaction between A and B:

$$SS_{AB} = S \times \sum_{a=1}^{A} \sum_{b=1}^{B} (M_{ab} - M_{a..} - M_{.b} + M_{...})^2$$

$$= 3 \times [(4 - 3 - 4 + 3)^2 + (4 - 3 - 4 + 3)^2 \\
+ (2 - 3 - 2 + 3)^2 + (2 - 3 - 2 + 3)^2]$$

$$= 3 \times (0 + 0 + 0 + 0)$$

$$= 0.00$$

- Sum of squares within:

$$SS_{S(AB)} = \sum_{a=1}^{A} \sum_{b=1}^{B} \sum_{s=1}^{S} (Y_{abs} - M_{ab})^2$$

$$= (4 - 4)^2 + (4 - 4)^2 + (4 - 4)^2 + (1 - 2)^2 + (3 - 2)^2 + (2 - 2)^2 + \\
(5 - 4)^2 + (4 - 4)^2 + (3 - 4)^2 + (2 - 2)^2 + (2 - 2)^2 + (2 - 2)^2$$

$$= 0 + 0 + 0 + 1 + 1 + 0 + 1 + 0 + 1 + 0 + 0 + 0$$

$$= 4.00$$

10.1.6 Mean squares

- Mean square for the main effect of A:

$$MS_A = \frac{SS_A}{df_A} = \frac{0.00}{1} = 0.00$$

- Mean square for the main effect of B:

$$MS_B = \frac{SS_B}{df_B} = \frac{12.00}{1} = 12.00$$

- Mean square for the interaction between A and B:

$$MS_{AB} = \frac{SS_{AB}}{df_{AB}} = \frac{0.00}{1} = 0.00$$

- Mean square within:

$$MS_{S(AB)} = \frac{SS_{S(AB)}}{df_{S(AB)}} = \frac{4.00}{8} = \frac{1}{2} = 0.50$$
10.1.7 F ratios

- F ratio for source A:
  \[ F_A = \frac{MS_A}{MS_{S(AB)}} = \frac{0.00}{0.50} = 0.00 \]

- F ratio for source B:
  \[ F_B = \frac{MS_B}{MS_{S(AB)}} = \frac{12.00}{0.50} = 24.00 \]

- F ratio for source AB:
  \[ F_{AB} = \frac{MS_{AB}}{MS_{S(AB)}} = \frac{0.00}{0.50} = 0.00 \]

10.1.8 The ANOVA table

The computed quantities are summarized in an ANOVA table as follows:

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>12.00</td>
<td>12.00</td>
<td>24.00</td>
</tr>
<tr>
<td>AB</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>S(AB)</td>
<td>8</td>
<td>4.00</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>16.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10.1.9 Critical values of F

The critical values of F for any effect you are interested in, the main effect of A, the main effect of B and the interaction between A and B are found in the F table.

For this experiment, we found in the table the following values:

- For the main effect of A, since \( \nu_1 = 1 \) and \( \nu_2 = 8 \), the critical values of F at \( \alpha = .05 \) and \( \alpha = .01 \) are respectively 5.32 and 11.26.

- For the main effect of B, since \( \nu_1 = 1 \) and \( \nu_2 = 8 \), the critical values of F at \( \alpha = .05 \) and \( \alpha = .01 \) are respectively 5.32 and 11.26.

- For the interaction between A and B, since \( \nu_1 = 1 \) and \( \nu_2 = 8 \), the critical values of F at \( \alpha = .05 \) and \( \alpha = .01 \) are respectively 5.32 and 11.26.
10.1.10 Interpretation of the results

The statistical significance of the results for a 2-factor design are determined in the same manner as for a 1-factor design, for any effect you are interested in: the main effect of $A$, the main effect of $B$ or the interaction between $A$ and $B$.

We can reach the following conclusions:

- For the main effect of $A$, since the calculated value 0 of $F$ is less than both the critical values $F$, we cannot reject the null hypothesis and thus we need to suspend judgment.

- For the main effect of $B$, since the calculated value 24 of $F$ is larger than both the critical values $F$, we can reject the null hypothesis and accept the alternative hypothesis. So, we can conclude that 4th graders perform better than Ph.D students.

- For the interaction between $A$ and $B$, since the calculated value 0 of $F$ is less than both the critical values $F$, we cannot reject the null hypothesis and thus we need to suspend judgment.

10.1.11 Conclusion (APA style)

The gender of subjects had no effect on performance, $F(1, 8) = 0$, nor did gender interact with age $F(1, 8) = 0$. Age of subjects showed a significant effect on subject’s performance: 4th graders show better motor competence than Ph.D. student, $F(1, 8) = 24$, $MS_e = 0.5$, $p < .01$. 
10.2 Exercises

Exercise 10.1: Noisy Math

Zentall & Shaw (1980) designed an experiment to assess the effect of noise on the performance of children solving easy mathematical problems. Two groups of subjects were tested: the first one was composed of hyperactive students and the second one of non-hyperactive students. The task was performed under high- and low-noise conditions. In a replication of this experiment, we measured the number of problems solved and we obtained the set of scores listed in Table 10.2:

<table>
<thead>
<tr>
<th></th>
<th>High Noise</th>
<th>Low Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyperactive</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Control</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Group</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Group</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Group</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

In this experiment,

1. What are the independent variables and their levels?

   $A$
   
   $a_1$
   
   $a_2$

   $B$
   
   $b_1$
   
   $b_2$

2. What is the dependent variable?

   $Y$
3. What type of experimental design is this?

4. What are the null hypotheses?

5. What are the alternative hypotheses?
6. What are the degrees of freedom?
7. What are the different sums of scores?
8. What are the different means?

9. Plot the means.
10. Compute the different sums of squares.
11. Compute the mean squares.
12. Compute the $F$ ratios.
13. Complete the ANOVA table.

<table>
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<tr>
<th>Source</th>
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<td>_____</td>
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<td></td>
</tr>
</tbody>
</table>

14. Find the critical values for $F$.


15. Interpret the results.
16. Write the conclusion in APA style.
Exercise 10.2: J.G.P. . . . strikes again!

Another exciting set of data from the latest issue of the *Journal of Generic and Fictional Psychology*. We leave it to imaginative readers to supply their own “experimental” design. Here is a set of data to inspire you:

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$M_b$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>54</td>
<td>47</td>
<td>49</td>
</tr>
<tr>
<td>$b_2$</td>
<td>58</td>
<td>45</td>
<td>47</td>
</tr>
<tr>
<td>$b_3$</td>
<td>63</td>
<td>60</td>
<td>72</td>
</tr>
<tr>
<td>$M_a..$</td>
<td>55</td>
<td>45</td>
<td>$M_{..} = 50$</td>
</tr>
</tbody>
</table>

1. Complete the analysis of variance and fill in the ANOVA table.
<table>
<thead>
<tr>
<th>Source</th>
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<th></th>
<th></th>
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</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Build a new set of data like this one but with no effect of A.
3. Build a new set of data like this one but with no effect of $B$. 
10.2 Exercises
4. Build a new set of data like this one but with no effect of $AB$. 
5. Build a new set of data like this one but with no effect of $A$ and no effect of $B$. 
6. Build a new set of data like this one but with no effect of A and no effect of AB.
Exercise 10.3: The Younger Jung (1903)

Still from Spitzer (1999, p. 222). Carl Gustav Jung, in his early years, was an experimental psychologist interested in what we call nowadays: Memory and Cognition (nobody’s perfect!). At that time (from 1902 till 1909) he was a student of Bleuler (a very famous psychologist of a stature equivalent to A. Binet or W. James) in Zurich. He theorized that the effect observed by Aschaffenburg (1899) was due to a weakening of selective attention rather than fatigue per se (i.e., weakening of attention was confounded with fatigue). To support his hypothesis, he had subjects generating word associations under two different conditions: A control condition and a “diverted attention” condition. In this last condition, subjects had to draw lines at various speeds following the beat of a metronome (so they were doing two things at the same time which divides attention). Jung recorded the word associations and tabulated them as being conceptual or pun-ish. When re-analyzing the data of Jung, Spitzer (1999) considers that the type of association observed is an independent variable. In this “nice numbers” rendition of this experiment, 5 subjects were used per condition. The data are given in Table 10.3.

Suppose that the design is an $S(A \times B)$ design (which it is not).

1. Plot the results of this experiment.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Normal Attention</th>
<th>Divided Attention</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conceptual</td>
<td>Pun-ish</td>
</tr>
<tr>
<td>1</td>
<td>116</td>
<td>51</td>
</tr>
<tr>
<td>2</td>
<td>125</td>
<td>51</td>
</tr>
<tr>
<td>3</td>
<td>95</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>107</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>107</td>
<td>21</td>
</tr>
</tbody>
</table>

$M_{ab}$. 110 30 80 60
2. Answer the following questions:

- Is there an effect of the type of association?
- Is there an effect of the attention condition?
- And do these variables interact?
3. What pairs of means (if any) show a reliable difference?
4. Looking at plot of the group means I have the impression that the experimental effect has a rotated V-shape (a shape like this: >). How much does that idea fit the group results? How much does that idea fit the individual results? Are these conclusions reliable?
11

ANOVA One Factor Repeated Measures, $S \times A$

11.1 Summary

In an *independent measurement*, or *between subjects* design, a subject participates in only one condition of the experiment. In a *repeated measurement*, or *within subjects* design, a subject participates in all the conditions of the experiment. In other words, the same subjects appear in all the experimental groups. This difference between independent and repeated measurement is illustrated in the following tables.

### Between-Subject Design

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
<td>Bob</td>
<td>Ted</td>
</tr>
<tr>
<td>Tom</td>
<td>Jay</td>
<td>Joe</td>
</tr>
<tr>
<td>Toto</td>
<td>Bill</td>
<td>John</td>
</tr>
</tbody>
</table>

### Within-Subject Design

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
<td>Jim</td>
<td>Jim</td>
</tr>
<tr>
<td>Tom</td>
<td>Tom</td>
<td>Tom</td>
</tr>
<tr>
<td>Toto</td>
<td>Toto</td>
<td>Toto</td>
</tr>
</tbody>
</table>

11.1.1 Partition of the total sum of squares

In an $S(A)$ design, the total sum of squares was partitioned into two parts:

$$SS_{total} = SS_A + SS_{S(A)}$$
where $SS_A$, or variability between groups, expresses the effect of the independent variable and $SS_{S(A)}$, or variability within groups, represents the experimental error.

In an $S \times A$ design, the variability within groups can be subdivided further into two parts: the variability due to the subjects, denoted $SS_S$ and the error or residual variability, denoted $SS_{AS}$. Therefore, the total sum of squares can be partitioned into three parts:

$$SS_{total} = SS_A + SS_S + SS_{AS}$$

**effect of $A$** + **effect of $S$** + **experimental error**

### 11.1.2 The criterion $F$

The criterion $F$ is similar to the one used in a between-subjects design with the exception that the variability between groups is now divided by the residual variability (instead of by the within groups variability):

$$F = \frac{\text{variability between groups}}{\text{residual variability}}$$

### 11.1.3 How to compute the $F$ ratio

- **Sums of squares:**
  - Sum of squares between groups
    $$SS_A = S \sum (M_{a} - M_{..})^2$$
  - Sum of squares between subjects
    $$SS_S = A \sum (M_{s} - M_{..})^2$$
  - Sum of squares residual
    $$SS_{AS} = \sum (Y_{as} - M_{a} - M_{s} + M_{..})^2$$

- **Degrees of freedom:**
  - The total number of degrees of freedom is:
    $$df_{total} = (A \times S) - 1$$
  - The number of degrees of freedom between groups is:
    $$df_A = A - 1$$
– The number of degrees of freedom _between subjects_ is:

\[ df_S = S - 1 \]

– The number of degrees of freedom _residual_ is:

\[ df_{AS} = (A - 1)(S - 1) \]

*Note* that: \( df_{total} = df_A + df_S + df_{AS} \)

• Mean squares:

  – Mean square _between groups_

\[ MS_A = \frac{SS_A}{df_A} \]

  – Mean square _between subjects_

\[ MS_S = \frac{SS_S}{df_S} \]

  – Mean square _residual_

\[ MS_{AS} = \frac{SS_{AS}}{df_{AS}} \]

• The _F_ index:

\[ F_A = \frac{MS_A}{MS_{AS}} \]

11.1.4 **Example: \( S \times A \) design**

In an experiment on paired-associate learning, four randomly chosen subjects were presented with two different lists of 35 pairs of words to learn. The first list was composed of concrete words, the second list of abstract words. Each subject was successively asked to learn and recall the two lists. The order of presentation of the lists was counterbalanced. The dependent variable is the number of words recalled. The data for this experiment are presented in Table 11.1.

• Sums of squares:

  – Sum of squares _between groups_

\[ SS_A = S \sum (M_{a.} - M_{..})^2 \]

\[ = 4 \times [(26 - 20)^2 + (14 - 20)^2] \]

\[ = 4 \times [(6)^2 + (-6)^2] \]
### Summary

#### TABLE 11.1 Data for the $S \times A$ design example

<table>
<thead>
<tr>
<th>Subject</th>
<th>Concrete</th>
<th>Abstract</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26</td>
<td>10</td>
<td>$M_1 = 18$</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>16</td>
<td>$M_2 = 19$</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>12</td>
<td>$M_3 = 21$</td>
</tr>
<tr>
<td>4</td>
<td>26</td>
<td>18</td>
<td>$M_4 = 22$</td>
</tr>
</tbody>
</table>

$M_a = 26$  
$M_{1.} = 14$  
$M_{..} = 20$

\[
\begin{align*}
M_a, & = 4 \times [36 + 36] \\
& = 4 \times 72 \\
& = 288.00
\end{align*}
\]

- **Sum of squares between subjects**

\[
SS_S = A \sum (M_s - M_{..})^2
= 2 \times [(18 - 20)^2 + (19 - 20)^2 + (21 - 20)^2 + (22 - 20)^2]
= 2 \times [(-2)^2 + (-1)^2 + (1)^2 + (2)^2]
= 2 \times [4 + 1 + 1 + 4]
= 2 \times 10
= 20.00
\]

- **Sum of squares residual**

\[
SS_{AS} = \sum (Y_{as} - M_a, - M_s + M_{..})^2
= (26 - 26 - 18 + 20)^2 + (22 - 26 - 19 + 20)^2
+ (30 - 26 - 21 + 20)^2 + (26 - 26 - 22 + 20)^2
+ (10 - 14 - 18 + 20)^2 + (16 - 14 - 19 + 20)^2
+ (12 - 14 - 21 + 20)^2 + (18 - 14 - 22 + 20)^2
= 4^2 + (-3)^2 + (3)^2 + (-2)^2 + (-2)^2 + (3)^2 + (-3)^2 + (2)^2
= 4 + 9 + 9 + 4 + 4 + 9 + 9 + 4
= 52.00
\]

- **Degrees of freedom:**

- The number of degrees of freedom *between groups* is:

\[
df_A = A - 1 = 2 - 1 = 1
\]
- The number of degrees of freedom between subjects is:
  \[ df_S = S - 1 = 4 - 1 = 3 \]
- The number of degrees of freedom residual is:
  \[ df_{AS} = (A - 1)(S - 1) = (2 - 1)(4 - 1) = (1)(3) = 3 \]

- Mean squares:
  - Mean square between groups
    \[ MS_A = \frac{SS_A}{df_A} = \frac{288.00}{1} = 288.00 \]
  - Mean square between subjects
    \[ MS_S = \frac{SS_S}{df_S} = \frac{20.00}{3} = 6.66 \]
  - Mean square residual
    \[ MS_{AS} = \frac{SS_{AS}}{df_{AS}} = \frac{52.00}{3} = 17.33 \]

- The \( F \) index:
  \[ F_A = \frac{MS_A}{MS_{AS}} = \frac{288.00}{17.33} = 16.62 \]

- Finally, we can fill in the ANOVA table:

<table>
<thead>
<tr>
<th>Source</th>
<th>( df )</th>
<th>( SS )</th>
<th>( MS )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>288.00</td>
<td>288.00</td>
<td>16.62</td>
</tr>
<tr>
<td>S</td>
<td>3</td>
<td>20.00</td>
<td>6.66</td>
<td>—</td>
</tr>
<tr>
<td>AS</td>
<td>3</td>
<td>52.00</td>
<td>17.33</td>
<td>—</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>360.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The critical value found in the \( F \) table, for \( \nu_1 \) equal to 1 and \( \nu_2 \) equal to 3, is 10.13 for \( \alpha = .05 \). Therefore at the .05 level our calculated \( F \) of 16.62 is larger than the table value and we can reject the null hypothesis that the type of words (concrete vs. abstract) makes no difference in the number of pairs correctly recalled. In other words, at the .05 level we can accept the alternative hypothesis and conclude that concrete words are better recalled than abstract words, \( F(1, 3) = 16.62, MS_e = 17.33, p < .05 \). Note that at \( \alpha = .01 \) the critical value is equal to 34.12, and hence that for that significance level we cannot reject the null hypothesis.
11.2 Exercises

Exercise 11.1: Using the “dot”

In a longitudinal study on the acquisition of punctuation marks by elementary school children, a developmental psychologist asked a group of 4 children to write a narrative at the beginning of each academic years during 3 consecutive years (from 2nd through 4th grade). For each narrative, she recorded the number of punctuation marks used.

<table>
<thead>
<tr>
<th>Subject</th>
<th>2nd grade</th>
<th>3rd grade</th>
<th>4th grade</th>
<th>$M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>9</td>
<td>17</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>12</td>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>7</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>8</td>
<td>17</td>
<td>11</td>
</tr>
</tbody>
</table>

$\bar{M}_a$. 8 9 16 11

1. Indicate the following:

- Independent variable and its levels.

- Dependent variable.
11.2 Exercises

• Experimental design.

• Null hypothesis.

• Alternative hypothesis.

2. Plot the results of this experiment.
3. Indicate the numbers of degrees of freedom:

- between groups

- between subjects
• residual

4. Compute the sum of squares.

• between groups

• between subjects
5. Compute the mean squares.

- between groups

- between subjects
11.2 Exercises

- residual

6. Compute the index $F$. 
7. Complete the ANOVA table.

<table>
<thead>
<tr>
<th>Source</th>
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<td>Total</td>
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<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

8. Interpret the results of the experiment.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

9. Write the conclusion using APA style.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

10. Can we say that the number of punctuation marks increases as a linear function of age?
Exercise 11.2: The French Connection . . .

This story is taken from Fayol and Abdi (1986, data made nice from Abdi, 1987, p. 329 ff.). A long time ago, we studied how punctuation marks and connectors link sentences or propositions. A connector (as the name indicates) is a word connecting two sentences (e.g., and, then, after). And a punctuation mark is, well, a punctuation mark. Because both punctuation marks and connectors serve to link two sentences, we refer to them as links. In one experiment, we wanted to explore the conjoint effect of connectors and punctuation marks. We decided to use four different connectors: $\emptyset$ (i.e., no connector), “and”, “then”, and “after.” We used three different punctuation marks: $\emptyset$ (i.e., no punctuation), “comma,” and “period.” Because we wanted always to have something linking the two sentences, we obtained eleven pairs of links: 4 connectors $\times$ 3 punctuation marks minus 1 (for the “$\emptyset$ and $\emptyset$” condition that would be without any link).

In this nice number replication, we asked 21 subjects to give a score on a scale from 0 to 50 to pairs of sentences. A score of 0 means that the sentences are completely unrelated, a score of 50 means the sentences are as related as they can be. Each subject saw 110 such pairs of sentences (10 sentences for each possible combinations of links). The association between sentences and pairs of links was randomized for each subject (Question: why did we do that?). We decided to analyze, as a dependent variable, the average score given by each subject for each link. As an interesting piece of information (this is a hint by the way), we noted that the value of the subject mean square was equal to $MS_s = 1,240.00$.

The authors of that superb experiment$^1$ had several predictions detailed below:

- Punctuation marks “link” better than connectors.
- The comma links better than period.
- “After” links better than “then” or “and.”
- “And” links better than “then.”

1. What are the independent and dependent variables?

$^1$Oooops. Sorry, I couldn't resist!
TABLE 11.2 Data inspired from Abdi (1987, p. 329, originally from Fayol & Abdi, 1986). The numbers in parentheses are the group variances.

<table>
<thead>
<tr>
<th>Connector</th>
<th>None</th>
<th>And</th>
<th>Then</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Punctuation</strong></td>
<td>None</td>
<td>40</td>
<td>33</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(160)</td>
<td>(210)</td>
<td>(220)</td>
</tr>
<tr>
<td><strong>Comma ,</strong></td>
<td>34</td>
<td>34</td>
<td>36</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td>(200)</td>
<td>(200)</td>
<td>(190)</td>
<td>(160)</td>
</tr>
<tr>
<td><strong>Mark</strong></td>
<td>Period .</td>
<td>29</td>
<td>33</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>(250)</td>
<td>(210)</td>
<td>(230)</td>
<td>(210)</td>
</tr>
</tbody>
</table>

2. What is the experimental design?

3. What are the experimental and non-experimental factors?
4. Plot the results and analyze this experiment.
Exercise 11.3: Once (again!) upon a time in Aschaffenburger!

Story still taken from Spitzer (1999, p. 220). Aschaffenburg (1899, no typo here, the work is one century old), was a German (as the name suggests\(^2\)) psychologist. To explore the effect of fatigue on cognition, he used the technique of free association where subjects associate an answer word to a stimulus word. He categorized an association as conceptual when the answer and the stimulus are related by their meaning (e.g., answer variance for stimulus mean); he categorized an association as “pun-ish\(^3\)” (or following Spitzer’s terminology as clang) when the answer and the stimulus are related by their sound (e.g., answer queen for stimulus mean).

His idea was that, when subjects are tired, their word associations should be more pun-ish (or less conceptual). In order to explore this hypothesis, Aschaffenburg used subjects working on night shifts. The same five subjects gave word associates to list made of one hundred words at four different times of the night: 9PM, midnight, 3AM and 6AM. The dependent variable was the number of pun-ish associations (over a possible maximum of 100).

He obtains the data given in Table 11.3.

<table>
<thead>
<tr>
<th>Subject</th>
<th>9PM</th>
<th>Midnight</th>
<th>3AM</th>
<th>6AM</th>
<th>M.s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>22</td>
<td>24</td>
<td>34</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>22</td>
<td>20</td>
<td>32</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>10</td>
<td>17</td>
<td>29</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>9</td>
<td>11</td>
<td>21</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>12</td>
<td>13</td>
<td>19</td>
<td>13</td>
</tr>
</tbody>
</table>

\(M_a,\) 9 15 17 27

1. Plot the results of this experiment.

\(^2\)I am not kidding this is a real name!

\(^3\)Isn’t it funny?
2. Is there an effect of the time of the night on the number of pun-ish associations?
3. What are the pairs of conditions that show a reliable difference?
4. When looking at the results, I have the impression that the average number of pun-ish associations increases as a linear function of the time of the night. What contrast coefficients should I use to test that idea?
5. How well do I predict the group results with this linear hypothesis?
6. How well do I predict the individual results with this linear hypothesis?
7. Are these predictions reliable?
8. Create an example with the same error mean square but no effect of $A$ at all.
11.2 Exercises
11.2 Exercises
9. Create an example with the same error mean square and a small effect of $A$ which is *not* statistically reliable.
12

ANOVA Partially Repeated Measures, $S(A) \times B$

12.1 Summary

- In an **independent measurement**, or **between subjects** design, a subject participates in only one condition of the experiment.

- In a **repeated measurement**, or **within subjects** design, a subject participates in all the conditions of the experiment. In other words, the same subjects appear in all the experimental groups.

- **Partially repeated measures** designs, also sometimes called mixed designs, include both of these ways of assigning subjects to experimental conditions. In a two factor partially repeated design, one factor is between subjects and the other factor is within subjects.

This design is symbolized by the notation $S(A) \times B$. The factor $A$ is the between subject variable and the factor $B$ is the within or repeated subject variable. Different subjects are assigned to the levels of $A$, but all subjects participate in all the levels of $B$. Therefore, subjects are nested in the independent variable $A$ and crossed with the independent variable $B$.

The difference between independent, repeated measurement, and partially repeated measurement is illustrated in the following Tables.

<table>
<thead>
<tr>
<th>One-Factor Between-Subject Design $S(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition 1</td>
</tr>
<tr>
<td>Jim</td>
</tr>
<tr>
<td>Tom</td>
</tr>
<tr>
<td>Toto</td>
</tr>
</tbody>
</table>
### One-Factor Within-Subject Design $S \times A$

<table>
<thead>
<tr>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
<td>Jim</td>
<td>Jim</td>
</tr>
<tr>
<td>Tom</td>
<td>Tom</td>
<td>Tom</td>
</tr>
<tr>
<td>Toto</td>
<td>Toto</td>
<td>Toto</td>
</tr>
</tbody>
</table>

### Two-Factor Partially Repeated Measures Design $S(A) \times B$

<table>
<thead>
<tr>
<th>Factor $A$</th>
<th>Condition $b_1$</th>
<th>Condition $b_2$</th>
<th>Condition $b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition $a_1$</td>
<td>Jim</td>
<td>Jim</td>
<td>Jim</td>
</tr>
<tr>
<td></td>
<td>Tom</td>
<td>Tom</td>
<td>Tom</td>
</tr>
<tr>
<td></td>
<td>Toto</td>
<td>Toto</td>
<td>Toto</td>
</tr>
<tr>
<td>Condition $a_2$</td>
<td>Bob</td>
<td>Bob</td>
<td>Bob</td>
</tr>
<tr>
<td></td>
<td>Jay</td>
<td>Jay</td>
<td>Jay</td>
</tr>
<tr>
<td></td>
<td>Bill</td>
<td>Bill</td>
<td>Bill</td>
</tr>
<tr>
<td>Condition $a_3$</td>
<td>Ted</td>
<td>Ted</td>
<td>Ted</td>
</tr>
<tr>
<td></td>
<td>Joe</td>
<td>Joe</td>
<td>Joe</td>
</tr>
<tr>
<td></td>
<td>John</td>
<td>John</td>
<td>John</td>
</tr>
</tbody>
</table>

#### 12.1.1 Hypotheses

As for the two factor between-subjects design there are three null hypotheses: one for the factor $A$, one for $B$, and one for the interaction $AB$. Likewise, there are three alternative hypotheses: one for the factor $A$, one for $B$, and one for the interaction $AB$.

#### 12.1.2 Partition of the total sum of squares

The total sum of squares can be partitioned into five parts:

$$SS_{\text{total}} = SS_A + SS_B + SS_{AB} + SS_{S(A)} + SS_{BS(A)}$$

#### 12.1.3 The $F$ criteria

The criteria for the $F$ ratios are similar to the ones used in a two factor between-subjects design, with the exception of the mean squares used in the denominators. The mean square between groups, $MS_A$, is now divided by the mean square of subjects nested in $A$, $MS_{S(A)}$, while the mean square
of the within subject factor $B$, $MS_B$, and the mean square of interaction, $MS_{AB}$, are both divided by the mean square of the interaction between $B$ and $S(A)$, $MS_{BS(A)}$.

### 12.1.4 How to compute the $F$ ratio

1. The following means will be needed:
   - $M_{..}$ — the grand mean
   - $M_{a..}$ — the mean for each level of $A$
   - $M_{..b}$ — the mean for each level of $B$
   - $M_{a..b}$ — the mean for each combination of level $A$ and $B$
   - $M_{a.s}$ — the mean for each subject

2. Degrees of Freedom:
   - Degrees of freedom total:
     \[
     df_{\text{total}} = (A \times B \times S) - 1
     \]
   - Degrees of freedom between subjects:
     \[
     df_A = A - 1
     \]
   - Degrees of freedom within subjects:
     \[
     df_B = B - 1
     \]
   - Degrees of freedom interaction:
     \[
     df_{AB} = (A - 1)(B - 1)
     \]
   - Degrees of freedom residual for $A$:
     \[
     df_{S(A)} = A(S - 1)
     \]
   - Degrees of freedom residual for $B$ and interaction $AB$:
     \[
     df_{BS(A)} = A(B - 1)(S - 1)
     \]

   Note that: $df_{\text{total}} = df_A + df_B + df_{AB} + df_{S(A)} + df_{BS(A)}$

3. Sums of squares:
• Sum of squares **between subjects**

\[ SS_A = BS \sum (M_{a..} - M_{...})^2 \]

• Sum of squares **within subjects**

\[ SS_B = AS \sum (M_{b.} - M_{...})^2 \]

• Sum of squares **interaction**

\[ SS_{AB} = S \sum (M_{ab.} - M_{a..} - M_{b.} + M_{...})^2 \]

• Sum of squares **residual** for factor A

\[ SS_{S(A)} = B \sum (M_{a.s} - M_{a..})^2 \]

• Sum of squares **residual** for factor B and interaction AB

\[ SS_{BS(A)} = \sum (Y_{abs} - M_{ab.} - M_{a.s} + M_{a..})^2 \]

4. Mean squares:

• Mean square **between subject factor A**

\[ MS_A = \frac{SS_A}{df_A} \]

• Mean square **within subject factor B**

\[ MS_B = \frac{SS_B}{df_B} \]

• Mean square **interaction** of A and B

\[ MS_{AB} = \frac{SS_{AB}}{df_{AB}} \]

• Mean square **test for factor A**

\[ MS_{S(A)} = \frac{SS_{S(A)}}{df_{S(A)}} \]

• Mean square **test for factor B and interaction AB**

\[ MS_{BS(A)} = \frac{SS_{BS(A)}}{df_{BS(A)}} \]
5. The $F$ indices:

\[ F_A = \frac{MS_A}{MS_{S(A)}} \]
\[ F_B = \frac{MS_B}{MS_{BS(A)}} \]
\[ F_{AB} = \frac{MS_{AB}}{MS_{BS(A)}} \]

6. Critical values for the $F$ indices:

The critical value for each $F$ is determined in the usual manner. The degrees of freedom corresponding to the numerator of the $F$ ratio are the $\nu_1$ $F$ table parameters, and the degrees of freedom of the denominator are the $\nu_2$ parameters.
12.2 Exercises

Exercise 12.1: The other-race effect

A well-known phenomenon in the face recognition literature is the so-called “other-race” effect: People are better at recognizing faces of their own races than faces of other races. This effect was tested in the following (fictitious) experiment. Eight subjects participated in the experiment. Four of these subjects were Caucasian and four were Japanese. Each subject studied a set of face pictures. One-half of the pictures were of Caucasian faces and the other half were Japanese faces. After completing a distractor task that lasted for 10 minutes, the subjects were presented with a 2AFC test (2 Alternative Forced Choice test). In this test format, subjects have to decide which of the two faces (displayed on a computer screen) they have seen during the learning phase of the experiment. The dependent variable was the number of faces correctly recognized.

The results of this experiment are given in Table 12.1.

<table>
<thead>
<tr>
<th>Subject Race</th>
<th>Face Race</th>
<th>Caucasian</th>
<th>Japanese</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caucasian</td>
<td></td>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>$S_1$</td>
<td></td>
<td>34</td>
<td>8</td>
</tr>
<tr>
<td>$S_2$</td>
<td></td>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>$S_3$</td>
<td></td>
<td>39</td>
<td>3</td>
</tr>
<tr>
<td>Japanese</td>
<td></td>
<td>16</td>
<td>28</td>
</tr>
<tr>
<td>$S_1$</td>
<td></td>
<td>13</td>
<td>33</td>
</tr>
<tr>
<td>$S_2$</td>
<td></td>
<td>17</td>
<td>23</td>
</tr>
<tr>
<td>$S_3$</td>
<td></td>
<td>18</td>
<td>28</td>
</tr>
</tbody>
</table>

Perform an ANOVA on this data using the following steps and $\alpha = .05$.

1. Indicate the following:

   - Independent variables and their levels.
• Dependent variable.

• Experimental design.

• Null hypothesis.

• Alternative hypothesis.
2. Compute the means:
   - $M_{..}$ — the grand mean.
   - $M_{a..}$ — the mean for each level of $A$.
   - $M_{b.}$ — the mean for each level of $B$.
   - $M_{ab.}$ — the mean for each combination of level $A$ and $B$.
   - $M_{a.s}$ — the mean for each subject.
3. Indicate the number of degrees of freedom:

- Degrees of freedom total.

- Degrees of freedom between subjects.

- Degrees of freedom within subjects.

- Degrees of freedom interaction.

- Degrees of freedom residual for $A$. 
• Degrees of freedom residual for B and interaction AB.

4. Compute the sums of squares:

• Sum of squares between subjects.

• Sum of squares within subjects.

• Sum of squares interaction.
• Sum of squares *residual* for factor $A$.

• Sum of squares *residual* for factor $B$ and interaction $AB$.

5. Compute the mean squares:

• Mean square *between subject factor* $A$.

• Mean square *within subject factor* $B$.

• Mean square *interaction* of $A$ and $B$.
6. The $F$ values:

7. Determine the critical value for each $F$: 
8. Fill in the ANOVA table:

9. Graph the results of the experiment.

10. Interpret the results of the experiment.
Exercise 12.2: Priming Schizophrenia

Data and story from Spitzer (1994, reported in Spitzer, 1999, p. 249ff). This experiment explores phonological priming in schizophrenic patients and normal controls. But first, what is priming? In a lexical decision task, subjects are asked to decide if a series of letters projected in a screen is a word or not. When the word to be detected (called the target) is preceded by a related word (called the prime), the time to identify the target is smaller when compared to the time to identify the target presented alone (or when preceded by an unrelated word). For example, it will take less time to recognize that “butter” is a word when it is preceded by “bread” than when it is preceded by “chair.” This effect is called priming. When the prime shortens the reaction time (RT) for the second word, we call this a facilitation priming, when the prime lengthens the reaction time for the second word, we call this an inhibitory priming. Priming is estimated by subtracting the reaction time when the target is preceded by a prime from the reaction time when the target is presented alone. With a formula:

\[ \text{Priming} = \text{RT for target alone} - \text{RT for primed target} \]

A positive value corresponds to a facilitation priming, a negative value corresponds to an inhibitory priming.

Spitzer was interested in phonological priming (i.e., the prime rhymes with the target, as in “house-mouse”). He also wanted to explore the usefulness of this paradigm to study individual differences. Specifically, he wanted to see if phonological priming would differ between schizophrenic patients and normal controls. Because schizophrenic patients have many more “pun-ish” associations than normal control (remember the Kling-Clang example?), Spitzer predicted that the time course of phonological priming should differ for schizophrenic patients and normal controls. In this nice number replication of Spitzer, we have 5 schizophrenic patients and 5 normal controls. For each subject, we have computed the average phonological priming effect in three lag conditions (i.e., the lag is the duration of the pause between the presentation of the prime and the target\(^1\)): 0ms., 200ms., and 500ms. The data are presented in Table 12.2 on the next page. The various means necessary for the computation of the analysis of variance are given. Please make sure you understand the use of the indices.

\(^1\)For aficionados: the technical term is “Stimulus Onset Asynchrony” abbreviated as SOA.
TABLE 12.2 Nice number replication of Spitzer (1994, from Spitzer, 1999, p. 250ff). Time course of phonological priming for five schizophrenic patients and five normal controls.

<table>
<thead>
<tr>
<th>Population (A)</th>
<th>Subj.</th>
<th>Duration of pause between prime and target words</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0ms.</td>
<td>200ms.</td>
</tr>
<tr>
<td>Normal</td>
<td>s₁</td>
<td>-7</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>s₂</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>s₃</td>
<td>-29</td>
<td>1</td>
</tr>
<tr>
<td>Controls</td>
<td>s₄</td>
<td>-32</td>
<td>-8</td>
</tr>
<tr>
<td></td>
<td>s₅</td>
<td>-42</td>
<td>-18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(M_{11} = -24)</td>
<td>(M_{12} = -6)</td>
</tr>
<tr>
<td>Schizophrenic</td>
<td>s₁</td>
<td>27</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>s₂</td>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>s₃</td>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>Patients</td>
<td>s₄</td>
<td>7</td>
<td>-6</td>
</tr>
<tr>
<td></td>
<td>s₅</td>
<td>-11</td>
<td>-9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(M_{21} = 4)</td>
<td>(M_{22} = 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M₁.</td>
<td>M₂.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-10</td>
<td>-2</td>
</tr>
</tbody>
</table>

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1. Write down the formula of the design.

2. Plot the results of this experiment.

3. Analyze the data and report the results APA style.
4. Suppose that I predicted that the time course of priming for the control group should be linear. How can I test this prediction? (Note: this is a tough question, it requires some thinking.)
Exercise 12.3: “But they all look alike . . .”

A well-known phenomenon in the face recognition literature is the so-called “other-race” effect: People are better at recognizing faces of their own race than faces of other races. And other-race faces often are reported as “all looking alike.” The researchers in this (fictitious, but quite interesting) experiment manipulated two independent variables: Race of the subjects participating in the experiment (Caucasian or Japanese); and Race of the face photos presented (Caucasian or Japanese). Eight subjects participated in the experiment. During the learning phase the subjects saw 40 face photos, half of which were Caucasian and the other half were Japanese. After completing a distractor task that lasted 10 minutes, the subjects were presented with a 2AFC test (2 Alternative Forced Choice test). In this test format, subjects have to decide which of the two faces (displayed on a computer screen) they have seen during the learning phase of the experiment. The dependent variable was the number of faces correctly recognized.

The authors (as fictitious as the paper!) gave a table of the experimental means, see Table 12.3.

<table>
<thead>
<tr>
<th>Face Race</th>
<th>Subject Race</th>
<th>Caucasian</th>
<th>Japanese</th>
</tr>
</thead>
<tbody>
<tr>
<td>Caucasian</td>
<td>$M_{11} = 32$</td>
<td>$M_{21} = 16$</td>
<td>$M_{1} = 24$</td>
</tr>
<tr>
<td>Japanese</td>
<td>$M_{12} = 4$</td>
<td>$M_{22} = 28$</td>
<td>$M_{2} = 16$</td>
</tr>
</tbody>
</table>

The following information is taken from the result section of the paper:

- There is an effect of the race of the subject, $F(1,6) = 8.00, p < .05$.
- There is a trend toward significance for the effect of the face race, $F(1,6) = 4.00, p = .09$.
- They found a strong interaction between the race of the subject and the race of the face, $F(1,6) = 25.00, p < .01$.

Don’t you have the impression that you have seen these data before?
1. Build back the ANOVA table.
2. Suppose that the experimenters were able to get hold of twenty people to take part in the experiment. Would this have resulted in rewriting the result section of the paper if they had exactly the same table of means? Answer this question supposing that the $F$ values are the same as in Question 1. In other words, compute a new ANOVA table.
3. Again, let us suppose that the experimenters were able to get hold of twenty people to take part in the experiment. Would this have resulted in rewriting the result section of the paper if they had exactly the same table of means? Answer this question supposing that the mean squares of error are the same as in Question 1. In other words, compute a new ANOVA table.