CONTRAST ANALYSIS

Hervé Abdi[*] & *Lynne Williams*

A standard analysis of variance (ANOVA) provides an F test, which is called an *omnibus test* because it reflects all possible differences between the means of the groups analyzed by the ANOVA. However, most experimenters want to draw conclusions more precise than "the experimental manipulation had an effect on the participants' behavior." Precise conclusions can be obtained from *contrast analysis* because a contrast expresses a specific question about the pattern of results of an ANOVA. Specifically, a contrast corresponds to a prediction precise enough to be translated into a set of numbers called *contrast coefficients,* which together reflect the prediction. The correlation between the contrast coefficients and the observed group means directly evaluates the similarity between the prediction and the results.

When performing a contrast analysis, one needs to distinguish whether the contrasts are *planned* or post hoc. Planned, or a priori, contrasts are selected before running the experiment. In general, they reflect the hypotheses the experimenter wanted to test, and there are usually *few* of them. Post hoc, or a posteriori (after the fact), contrasts are decided after the experiment has been run. The goal of a posteriori contrasts is to ensure that unexpected results are reliable.

When performing a planned analysis involving several contrasts, one needs to evaluate whether these contrasts are mutually orthogonal or not. Two contrasts are *orthogonal* when their contrast coefficients are uncorrelated (i.e., their coefficient of correlation is zero). The number of possible orthogonal contrasts is one less than the number of levels of the independent variable.

All contrasts are evaluated by the same general procedure. First, the contrast is formalized as a set of contrast coefficients (also called *contrast weights*). Second, a specific F ratio

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Address Correspondence to Hervé Abdi,
Program in Cognition and Neurosciences, MS: Gr.4.1,
The University of Texas at Dallas,
Richardson, TX 75080—3021 USA
E-mail: herve@utdallas.edu   https://personal.utdallas.edu/~herve.

(denoted F_ψ) is computed. Finally, the probability associated with F_ψ is evaluated. This last step changes with the type of analysis performed.

Research Hypothesis as a Contrast Expression

When a research hypothesis is precise, it is possible to express it as a contrast. A research hypothesis, in general, can be expressed as a shape, a configuration, or a rank ordering of the experimental means. In all these cases, one can assign numbers that will reflect the predicted values of the experimental means. These numbers are called *contrast coefficients* when their mean is zero. To convert a set of numbers into a contrast, it suffices to subtract their mean from each of them. Often, for convenience, contrast coefficients are expressed with integers.

For example, assume that for a four-group design, a theory predicts that the first and second groups should be equivalent, the third group should perform better than these two groups, and the fourth group should do better than the third with an advantage of twice the gain of the third over the first and the second. When translated into a set of ranks, this prediction gives

C_1	C_2	C_3	C_4	Mean
1	1	2	4	2

After subtracting the mean, we get the following contrast:

C_1	C_2	C_3	C_4	Mean
–1	–1	0	2	0

In case of doubt, a good heuristic is to draw the predicted configuration of results, and then to represent the position of the means by ranks.

A Priori Orthogonal Contrasts

For Multiple Tests

When several contrasts are evaluated, several statistical tests are performed on the same data set, and this increases the probability of a Type I error (i.e., rejection of the null hypothesis when it is true). In order to control the Type I error at the level of the set (also known as the *family*) of contrasts, one needs to correct the α level used to evaluate each contrast. This correction for multiple contrasts can be done with the use of the Šidák equation, the Bonferroni (also known as Boole, or Dunn) inequality, or the Monte Carlo method.

Šidák and Bonferroni

The probability of making *at least one* Type I error for a family of orthogonal (i.e., statistically independent) contrasts (C) is denoted $\alpha[PF]$, it is computed as

$$\alpha[PF] = 1 - \left(1 - \alpha[PC]\right)^{C}. \qquad (1)$$

Here, $\alpha[PF]$ is the Type I error for the family of contrasts and $\alpha[PC]$ is the Type I error *per* contrast. This equation can be rewritten as

$$\alpha[PC] = 1 - \left(1 - \alpha[PF]\right)^{1/C}. \qquad (2)$$

This formula, called the Šidák equation, shows how to correct the $[PC]$ values used for each contrast.

Because the Šidák equation involves a fractional power, one can use an approximation known as the Bonferroni inequality (obtained from the first term of a Taylor expension), which relates $\alpha[PC]$ to $\alpha[PF]$:

$$\alpha[PC] \approx \frac{\alpha[PF]}{C}. \qquad (3)$$

Šidák and Bonferroni are related by the inequality

$$\alpha[PC] = 1 - \left(1 - \alpha[PF]\right)^{1/C} \geq \frac{\alpha[PF]}{C}. \qquad (4)$$

These two equations give, in general, results very close to each other. As can be seen, the Bonferroni inequality is a pessimistic estimation. Consequently, Šidák should be preferred. However, the Bonferroni inequality is easier to compute and more well known and hence it is used and cited more often.

Monte Carlo

The Monte Carlo technique can also be used to correct for multiple contrasts. The Monte Carlo technique consists of running a simulated experiment many times using random data, with the aim of obtaining a pattern of results showing what would happen just on the basis of chance. This approach can be used to quantify $\alpha[PF]$, the inflation of Type I error due to multiple testing. Equation 1 can then be used to set $\alpha[PC]$ in order to control the overall value of the Type I error.

As an illustration, suppose that six groups with 20 observations per group are created with data randomly sampled from a normal population. By construction, the H_0 is true (i.e., all population means are equal). Now, construct five *independent contrasts* from these six groups. For each contrast, compute an F test. If the probability associated with the statistical index is smaller than $\alpha = .05$, the contrast is said to reach significance (i.e., $\alpha[PC]$ is used). Then have a computer redo the experiment a large number of times, here 100,000 times. In sum, there are 100,000 experiments, therefore 100,000 families of 5 contrasts each, and 5 × 100,000 = 500,000 contrasts total. The results of one such simulation are given in Table 1.

Table 1 shows that the H_0 is rejected for 24,969 contrasts out of the 500,000 contrasts actually performed (i.e., 5 contrasts × 100,000 experiments). From these data, an estimation of $\alpha[PC]$ is computed as

$$\alpha[PC] = \frac{\text{number of contrasts having reached significance}}{\text{total number of contrasts}} = \frac{24,969}{500,000} = .04994 . \quad (5)$$

This value falls very close to the theoretical value of $\alpha = .05$.

Table 1 Results of a Monte Carlo Simulation

Number of Families With X Type I Errors	*X: Number of Type 1 Errors per Family*	*Number of Type I Errors*
77,381	0	0
20,390	1	20,390
2,111	2	4,222
115	3	345
3	4	12
0	5	0
Total 100,000		Total 24,969

Notes: Numbers of Type I errors when performing $C = 5$ contrasts for 100,000 analyses of variance performed on a six-group design when the H_0 is true. For example, 2,111 families out of the 100,000 have two Type I errors. This gives 2 × 2,111 = 4,222 Type I errors.

It can be seen also that for 77,381 experiments, no contrast reached significance. Correspondingly, for 22,619 experiments (i.e., 100,000 – 77,381), at least one Type I error was made. From these data, $\alpha[PF]$ can be estimated as

$$\alpha[PC] = \frac{\text{number of families with at least 1 Type I error}}{\text{total number of families}} = \frac{22,919}{100,000} = .22619 . \quad (6)$$

This value falls close to the theoretical value given by Equation 1:

$$\alpha[PF] = 1 - \left(1 - \alpha[PC]\right)^{C} = 1 - (1 - .05)^{5} = .226\,.$$

Checking the Orthogonality of Two Contrasts

Two contrasts are orthogonal (or independent) if their contrast coefficients are uncorrelated. Contrast coefficients have zero sum (and therefore a zero mean). Therefore, two contrasts, whose A contrast coefficients are denoted $C_{a,1}$ and $C_{a,2}$, are orthogonal if and only if

$$\sum_{a=1}^{A} C_{a,i} C_{a,j} = 0. \qquad (7)$$

Computing Sum of Squares, Mean Square, and F

The sum of squares for a contrast can be computed with the C_a coefficients. Specifically, the sum of square of φ is computed

$$SS_{\psi} = \frac{S\left(\sum C_a M_{a.}\right)^2}{\sum C_a^2} \qquad (8)$$

where S is the number of subjects in a group and $M_{a.}$ is the mean of Group a.

Also, because the sum of squares for a contrast has one degree of freedom, it is equal to the mean square of effect for this contrast:

$$MS_{\psi} = \frac{SS_{\psi}}{df_{\psi}} = \frac{SS_{\psi}}{1} = SS_{\psi}\,. \qquad (9)$$

The F_{ψ} ratio for a contrast is now computed as

$$F_{\psi} = \frac{MS_{\psi}}{MS_{\text{error}}}\,. \quad (10)$$

Note, incidentally that because the numerator of F (i.e., MS_{φ}) has only one degree of freedom, its square root is equal to a t-statistics which is the statistic reported by some software.

Table 2 Data From a (Fictitious) Replication of an Experiment by Smith (1979).

	Experimental Context				
	Group 1	*Group 2*	*Group 3*	*Group 4*	*Group 5*
	Same	*Different*	*Imagery*	*Photo*	*Placebo*
	25	11	14	25	8
	26	21	15	15	20
	17	9	29	23	10
	15	6	10	21	7
	14	7	12	18	15
	17	14	22	24	7
	14	12	14	14	1
	20	4	20	27	17
	11	7	22	12	11
	21	19	12	11	4
$Y_{a.}$	180	110	170	190	100
$M_{a.}$	18	11	17	19	10
$M_{a.} - M_{..}$	3	-4	2	4	-5
$\Sigma\,(Y_{as} - M_{a.})^2$	218	284	324	300	314

Source: Adapted from Smith (1979) and Abdi et al. 2009.

Note: The dependent variable is the number of words recalled.

Evaluating F for Orthogonal Contrasts

Planned orthogonal contrasts are equivalent to independent questions asked to the data. Because of that independence, the current procedure is to act as if each contrast were the only contrast tested: This amounts to *not* correcting for multiple tests. This procedure gives maximum power to the test. Practically, the null hypothesis for a contrast is tested by computing an F ratio as indicated in Equation 10 and evaluating its p value using a Fisher sampling distribution with $\nu_1 = 1$ and ν_2 being the number of degrees of freedom of MS_{error} [e.g., in independent measurement designs with A groups and S observations per group, $\nu_2 = A(S - 1)$].

Example

This example is inspired by an experiment that Steven M. Smith ran 1979. The main purpose of this experiment was to show that one's being in the same mental context for learning and for testing leads to better performance than being in different contexts. During the learning phase, participants learned a list of 80 words in a room painted with an orange color, decorated with posters, paintings, and a quite a large amount of paraphernalia. At the end of the learning phase, a memory test was performed to give subjects the impression that the experiment was over. One day later, the participants were unexpectedly retested on their memory. An experimenter asked them to write down all the words from the list that they could remember. The test took place in five different experimental conditions. Fifty subjects (10 per group) were randomly assigned to one of the five experimental groups. The five experimental conditions were

1. *Same context.* Participants were tested in the same room in which they learned the list.
2. *Different context.* Participants were tested in a room very different from the one in which they learned the list. The new room was located in a different part of the campus, painted grey, and looked very austere.
3. *Imaginary context.* Participants were tested in the same room as participants from Group 2. In addition, they were told to try to remember the room in which they learned the list. In order to help them, the experimenter asked them several questions about the room and the objects in it.
4. *Photographed context.* Participants were placed in the same condition as Group 3, and in addition, they were shown photos of the orange room in which they learned the list.

5. *Placebo context.* Participants were in the same condition as participants in Group 2. In addition, before starting to try to recall the words, they are asked to perform a warm-up task, namely, to try to remember their living room.

The data and ANOVA results of the replication of Smith's experiment are given in Tables 2 and 3.

Table 3 ANOVA Table for a (Fictitious) Replication of Smith's Experiment (1979)

Source	*df*	*SS*	*MS*	*F*	*Pr(F)*
Experimental	4	700.00	175.00	5.469**	.00119
Error	45	1,440.00	32.00		
Total	49	21,400.00			

Source: Adapted from Smith (1979) and Abdi et al., (2009).

Note: $^{**}p < .01$.

Research Hypotheses for Contrast Analysis

Several research hypotheses can be tested with Smith's experiment. Suppose that the experiment was designed to test these hypotheses:

- *Research Hypothesis 1.* Groups for which the context at test matches the context during learning (i.e., is the same or is simulated by imaging or photography) will perform better than groups with different or placebo contexts.
- *Research Hypothesis 2.* The group with the same context will differ from the group with imaginary or photographed contexts.
- *Research Hypothesis 3.* The imaginary context group differs from the photographed context group.
- *Research Hypothesis 4.* The different context group differs from the placebo group.

Table 4 Orthogonal Contrasts for the Replication of Smith (1979)

Contrast	*Group 1*	*Group 2*	*Group 3*	*Group 4*	*Group 5*	*PCa*
ψ_1	+2	–3	+2	+2	–3	0
ψ_2	+2	0	–1	–1	0	0
ψ_3	0	0	+1	–1	0	0
ψ_4	0	+1	0	0	–1	0

Source: Adapted from Smith (1979).

Contrasts

The four research hypotheses are easily transformed into statistical hypotheses. For example, the first research hypothesis is equivalent to stating the following null hypothesis: The means of the population for Groups 1, 3, and 4 have the same value as the means of the population for Groups 2 and 5. This is equivalent to contrasting Groups 1, 3, and 4, on one hand, and Groups 2 and 5, on the other. This first contrast is denoted ψ_1:

$$\psi_1 = 2\mu_1 - 3\mu_2 + 2\mu_3 + 2\mu_4 + 3\mu_5 .$$

The null hypothesis to be tested is

$$H_{0,s1} : \psi_1 = 0.$$

This first contrast is equivalent to defining the following set of coefficients C_a:

$$\begin{array}{cccccc} \mathrm{Gr.1} & \mathrm{Gr.2} & \mathrm{Gr.3} & \mathrm{Gr.4} & \mathrm{Gr.5} & \sum_{a}^{A} C_a \\ +2 & -3 & +2 & +2 & -3 & 0 \end{array}$$

Note that the sum of the coefficients C_a is zero, as it should be for a contrast. Table 4 shows all four contrasts.

Are the Contrasts Orthogonal?

Now the problem is to decide whether the contrasts constitute an orthogonal family. We check that every pair of contrasts is orthogonal by using Equation 7. For example, Contrasts 1 and 2 are orthogonal because

$$\sum_{a=1}^{A} C_{a,1}C_{a,2} = (2\times 2)+(-3\times 0)+(2\times -1)+(2\times -1)+(-3\times 0)+(0\times 0) = 4-4 = 0 .$$

Table 5 Steps for the Computation of SSc1 of Smith (1979)

Group	$M_{a.}$	C_a	$C_aM_{a.}$	C^2_a
1	18.00	+2	+36.00	4
2	11.00	–3	–33.00	9
3	17.00	+2	+34.00	4
4	19.00	+2	+38.00	4
5	10.00	–3	–30.00	9
		0	45.00	30

Source: Adapted from Smith (1979).

F test

The sum of squares and F_ψ for a contrast are computed from Equations 8 and 10. For example, the steps for the computations of $SS_{\psi 1}$ are given in Table 5.

$$SS_{\psi 1} = \frac{S(\sum C_a M_{a.})^2}{\sum C_a^2} = \frac{10 \times (45.00)^2}{30} = 675.00$$

$$MS_{\psi 1} = 675.00$$

$$F_{\psi 1} = \frac{MS_{\psi 1}}{MS_{error}} = \frac{675.00}{32.00} = 21.094.$$

The significance of a contrast is evaluated with a Fisher distribution with 1 and $A(S-1) = 45$ degrees of freedom, which gives a critical value of 4.06 for $\alpha = 0.5$ (7.23 for $\alpha = .01$).

The sums of squares for the remaining contrasts are $SS_{\psi 2} = 0$, $SS_{\psi 3} = 20$, and $SS_{\psi 4} = 5$ with 1 and $A(S-1) = 45$ degrees of freedom. Therefore, ψ_2, ψ_3, and ψ_4 are nonsignificant. Note that the sums of squares of the contrasts add up to $SS_{experimental}$. That is,

$$SS_{experimental} = SS_{\psi 1} + SS_{\psi 2} + SS_{\psi 3} + SS_{\psi 4} = 675.00 + 0.00 + 20.00 + 5.00 = 700.00.$$

When the sums of squares are orthogonal, the degrees of freedom are added the same way as the sums of squares are. This explains why the maximum number of orthogonal contrasts is equal to the number of degrees of freedom of the experimental sum of squares.

A Priori Nonorthogonal Contrasts

So orthogonal contrasts are relatively straightforward because each contrast can be evaluated on its own. Nonorthogonal contrasts, however, are more complex. The main problem is to assess the importance of a given contrast conjointly with the other contrasts. There are currently two (main) approaches to this problem. The classical approach corrects for multiple statistical tests (e.g., using a Šidák or Bonferroni correction), but essentially evaluates each contrast as if it were coming from a set of orthogonal contrasts. The multiple regression (or modern) approach evaluates each contrast as a predictor from a set of nonorthogonal predictors and estimates its *specific* contribution to the explanation of the dependent variable. The classical approach evaluates each contrast for itself, whereas the multiple regression approach evaluates each contrast as a member of a set of contrasts and estimates the specific contribution of each contrast in this set. For an orthogonal set of contrasts, the two approaches are equivalent.

The Classical Approach

Some problems are created by the use of multiple nonorthogonal contrasts. The most important one is that the greater the number of contrasts, the greater the risk of a Type I error. The general strategy adopted by the classical approach to this problem is to correct for multiple testing.

Šidák and Bonferroni Corrections

When a family's contrasts are nonorthogonal, Equation 10 gives a lower bound for $\alpha[PC]$. So, instead of having the equality, the following inequality, called the Šidák inequality, holds:

$$\alpha[PF] \leq 1-\left(1-\alpha[PC]\right)^{C}. \qquad (12)$$

This inequality gives an upper bound for $\alpha[PF]$, and therefore the real value of $\alpha[PF]$ is smaller than its estimated value.

As earlier, we can approximate the Šidák inequality by Bonferroni as

$$\alpha[PF] < C\alpha[PC]. \qquad (13)$$

And, as earlier, Šidák and Bonferroni are linked to each other by the inequality

$$\alpha[PF] \leq 1-\left(1-\alpha[PC]\right)^{C} < C\alpha[PC]. \qquad (14)$$

Table 6 Nonorthogonal Contrasts for the Replication of Smith (1979)

Contrast	*Group 1*	*Group 2*	*Group 3*	*Group 4*	*Group 5*	*PCa*
ψ_1	2	–3	2	2	–3	0
ψ_2	3	3	–2	–2	–2	0
ψ_3	1	–4	1	1	1	0

Source: Adapted from Smith (1979).

Example

Let us go back to Smith's (1979) study (see Table 2). Suppose that Smith wanted to test these three hypotheses:

- *Research Hypothesis 1.* Groups for which the context at test matches the context during learning will perform better than groups with different contexts;
- *Research Hypothesis 2.* Groups with real contexts will perform better than those with imagined contexts;
- *Research Hypothesis 3.* Groups with any context will perform better than those with no context.

These hypotheses can easily be transformed into the set of contrasts given in Table 6. The values of F_{ψ} were computed with Equation 10 (see also Table 3) and are in shown in Table 7, along with their *p* values. If we adopt a value of $\alpha[PF]$ = .05, a Šidák correction will entail evaluating each contrast at the α level of $\alpha[PF]$ = .0170 (Bonferroni will give the approximate value of $\alpha[PF]$ = .0167). So, with a correction for multiple comparisons one can conclude that Contrasts 1 and 3 are significant.

Multiple Regression Approach

ANOVA and multiple regression are equivalent if one uses as many predictors for the multiple regression analysis as the number of degrees of freedom of the independent variable. An obvious choice for the predictors is to use a set of contrast coefficients. Doing so makes contrast analysis a particular case of multiple regression analysis. When used with a set of orthogonal contrasts, the multiple regression approach gives the same results as the ANOVA-based approach previously described. When used with a set of *nonorthogonal* contrasts, multiple regression quantifies the *specific* contribution of each contrast as the semi-partial coefficient of correlation between the contrast coefficients and the dependent variable. The multiple regression approach can be used for nonorthogonal contrasts as long as the following constraints are satisfied:

1. There are no more contrasts than the number of degrees of freedom of the independent variable.
2. The set of contrasts is linearly independent (i.e., not multicollinear). That is, no contrast can be obtained by combining the other contrasts.

Table 7 F_ψ Values for the Nonorthogonal Contrasts From the Replication of Smith (1979).

Contrast	$r_{Y\cdot}$	$r^2_{Y\cdot}$	*F*	*pF*
ψ_1	.9820	.9643	21.0937	<.0001
ψ_2	−.1091	.0119	0.2604	.6123
ψ_3	.5345	.2857	6.2500	.0161

Source: Adapted from Smith (1979).

Example

Let us go back once again to Smith's (1979) study of learning and recall contexts. Suppose we take our three contrasts (see Table 6) and use them as predictors with a standard multiple regression program. We will find the following values for the semi-partial correlation between the contrasts and the dependent variable:

$$\psi_1 : r^2_{Y.C_{a,1}|C_{a,2}C_{a,3}} = .1994$$
$$\psi_2 : r^2_{Y.C_{a,2}|C_{a,1}C_{a,3}} = .0000$$
$$\psi_3 : r^2_{Y.C_{a,3}|C_{a,1}C_{a,2}} = .0013,$$

with $r^2_{Y.C_{a,1}|C_{a,2}C_{a,3}}$ being the squared correlation of ψ_1 and the dependent variable with the effects of ψ_2 and ψ_3 partialled out. To evaluate the significance of each contrast, we compute an *F* ratio for the corresponding semipartial coefficients of correlation. This is done with the following formula:

$$F_{Y.C_{a,i}|C_{a,k}C_{a,\ell}} = \frac{r^2_{Y.C_{a,i}|C_{a,k}C_{a,\ell}}}{1-r^2_{Y.A}} \times df_{\text{residual}}. \qquad (15)$$

This results in the following F ratios for the Smith example:

$$\psi_1 : F_{Y.C_{a,1}|C_{a,2}C_{a,3}} = 13.3333, \quad p = .0007;$$
$$\psi_2 : F_{Y.C_{a,2}|C_{a,1}C_{a,3}} = 0.0000, \quad p = 1.0000;$$
$$\psi_3 : F_{Y.C_{a,3}|C_{a,1}C_{a,2}} = 0.0893, \quad p = .7665.$$

These F ratios follow a Fisher distribution with $\nu_1 = 1$ and $\nu_2 = 45$ degrees of freedom. F_{critical} = 4.06 when α = .05. In this case, ψ_1 is the only contrast reaching significance (i.e., with $F_\psi > F_{\text{critical}}$). The comparison with the classic approach shows the drastic differences between the two approaches.

A Posteriori Contrasts

For a posteriori contrasts, the family of contrasts is composed of all the possible contrasts even if they are not explicitly made. Indeed, because one chooses one of the contrasts to be made a posteriori, this implies that one has *implicitly* made and judged uninteresting *all* the possible contrasts that have not been made but *could* have been made. Hence, whatever the number of contrasts actually performed, the family is composed of all the possible contrasts. And, this number grows very fast: A conservative estimate indicates that the number of contrasts that can be made on A groups is equal to

$$1 + \{[(3^A - 1)/2] - 2^A\}. \qquad (16)$$

So, using a Šidàk or Bonferroni approach with a posteriori contrasts will, in general, not have enough power to be useful.

Scheffé's Test

Scheffé's test was devised to test all possible contrasts a posteriori while maintaining the overall Type I error level for the family at a reasonable level, as well as trying to have a conservative but relatively powerful test. The general principle is to ensure that no discrepant statistical decision can occur. A discrepant decision would occur if the omnibus test would fail to reject the null hypothesis, but one a posteriori contrast could be declared significant.

In order to avoid such a discrepant decision, the Scheffé approach first tests any contrast as if it were the largest possible contrast whose sum of squares is equal to the experimental sum of squares (this contrast is obtained when the contrast coefficients are equal to the deviations of the group means to their grand mean) and, second, makes the test of the largest contrast equivalent to the ANOVA omnibus test. So, if we denote by $F_{\text{critical,somnibus}}$ the critical value for the ANOVA omnibus test (performed on A groups), the largest contrast is equivalent to the omnibus test if its F_{ψ} is tested against a critical value equal to

$$F_{\text{critical,Scheffé}} = (A-1) \times F_{\text{critical,somnibus}}. \qquad (17)$$

Equivalently, F_{ψ} can be divided by (A 1), and its probability can be evaluated with a Fisher distribution with $\nu_1 = (A - 1)$ and ν_2 being equal to the number of degrees of freedom of the mean square error. Doing so makes it *impossible* to reach a discrepant decision.

4.1.1 An example: Scheffé

Suppose that the F_{ψ} ratios for the contrasts computed in Table 7 were obtained a posteriori. The critical value for the ANOVA is obtained from a Fisher distribution with $\nu_1 = A$ 1 = 4 and $\nu_2 = A(S-1) = 45$. For α = .05, this value is equal to $F_{\text{critical, omnibus}} = 2.58$. In order to evaluate whether any of these contrasts reaches significance, one needs to compare them to the critical value of

$$F_{\text{critical, Scheffé}} = (A-1) \times F_{\text{critical, omnibus}} = 4 \times 2.58 = 10.32\ . \qquad (18)$$

With Scheffé's approach, only the first contrast is considered significant.

Hervé Abdi and Lynne J. Williams

See also Analysis of Variance (ANOVA); Type I Error

Further Readings

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