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Contrast Analysis

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A standard analysis of variance (*a.k.a.* ANOVA) provides an F-test, which is called an *omnibus test* because it reflects all possible differences between the means of the groups analyzed by the ANOVA. However, most experimenters want to draw conclusions more precise than "the experimental manipulation has an effect on participants' behavior." Precise conclusions can be obtained from *contrast analysis* because a contrast expresses a specific question about the pattern of results of an ANOVA. Specifically, a contrast corresponds to a prediction precise enough to be translated into a set of numbers called *contrast coefficients* which reflect the prediction. The correlation between the contrast coefficients and the observed group means directly evaluates the similarity between the prediction and the results.

When performing a contrast analysis we need to distinguish whether the contrasts are *planned* or *post hoc*. *Planned* or *a priori* contrasts are selected before running the experiment. In general, they reflect the hypotheses the experimenter wanted to test and there are usually *few* of them. *Post hoc* or *a posteriori* (after the fact) contrasts are decided after the experiment has been run. The goal of *a posteriori* contrasts is to ensure that unexpected results are reliable.

When performing a planned analysis involving several contrasts, we need to evaluate if these contrasts are mutually orthogonal or not. Two contrasts are *orthogonal* when their contrast coefficients are uncorrelated (*i.e.,* their coefficient of correlation is zero). The number of possible orthogonal contrasts is one less than the number of levels of the independent variable.

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All contrasts are evaluated using the same general procedure. First, the contrast is formalized as a set of contrast coefficients (also called contrast weights). Second, a specific F ratio (denoted F_ψ) is computed. Finally, the probability associated with F_ψ is evaluated. This last step changes with the type of analysis performed.

1 How to express a research hypothesis as a contrast

When a research hypothesis is precise, it is possible to express it as a contrast. A research hypothesis, in general, can be expressed as a shape, a configuration, or a rank ordering of the experimental means. In all of these cases, we can assign numbers which will reflect the predicted values of the experimental means. These numbers are called *contrast coefficients* when their mean is zero. To convert a set of numbers into a contrast, it suffices to subtract their mean from each of them. Often, for convenience we will express contrast coefficients with integers.

For example, assume that for a 4-group design, a theory predicts that the first and second groups should be equivalent, the third group should perform better than these two groups and the fourth group should do better than the third with an advantage of twice the gain of the third over the first and the second. When translated into a set of ranks this prediction gives:

C_1	C_2	C_3	C_4	Mean
1	1	2	4	2

After subtracting the mean, we get the following contrast:

C_1	C_2	C_3	C_4	Mean
−1	−1	0	2	0

In case of doubt, a good heuristic is to draw the predicted configuration of results, and then to represent the position of the means by ranks.

2 A priori (planned) orthogonal contrasts

2.1 How to correct for multiple tests

When several contrast are evaluated, several statistical tests are performed on the same data set and this increases the probability of a Type I error (*i.e.,* rejecting the null hypothesis when it is true). In order to control the Type I error at the level of the set (*a.k.a.* the *family*) of contrasts one needs to correct the α level used to evaluate each contrast. This correction for multiple contrasts can be done using the Šidàk equation, the Bonferroni (*a.k.a.* Boole, or Dunn) inequality or the Monte-Carlo technique.

2.1.1 Šidàk and Bonferroni

The probability of making *as least one* Type I error for a family of orthogonal (*i.e.,* statistically *independent*) C contrasts is

$$\alpha[PF] = 1 - (1 - \alpha[PC])^C \quad . \tag{1}$$

with $\alpha[PF]$ being the Type I error for the family of contrasts; and $\alpha[PC]$ being the Type I error *per* contrast. This equation can be rewritten as

$$\alpha[PC] = 1 - (1 - \alpha[PF])^{1/C} \quad . \tag{2}$$

This formula, called the Šidàk equation, shows how to correct the $\alpha[PC]$ values used for each contrast.

Because the Šidàk equation involves a fractional power, ones can use an approximation known as the *Bonferroni* inequality, which relates $\alpha[PC]$ to $\alpha[PF]$ by

$$\alpha[PC] \approx \frac{\alpha[PF]}{C} \quad . \tag{3}$$

Šidàk and Bonferroni are related by the inequality

$$\alpha[PC] = 1 - (1 - \alpha[PF])^{1/C} \geq \frac{\alpha[PF]}{C} \quad . \tag{4}$$

They are, in general, very close to each other. As can be seen, the Bonferroni inequality is a pessimistic estimation. Consequently Šidàk should be preferred. However, the Bonferroni inequality is more well known, and hence, is used and cited more often.

2.1.2 Monte-Carlo

The Monte-Carlo technique can also be used to correct for multiple contrasts. The Monte Carlo technique consists of running a simulated experiment many times using random data,

Table 1: Results of a Monte-Carlo simulation. Numbers of Type I errors when performing $C = 5$ contrasts for $10,000$ analyses of variance performed on a 6 group design when the H_0 is true. How to read the table? For example, 192 families over $10,000$ have 2 Type I errors, this gives $2 \times 192 = 384$ Type I errors.

Number of families with X Type I errors	X: Number of Type 1 errors *per* family	Number of Type I errors
7,868	0	0
1,907	1	1,907
192	2	384
20	3	60
13	4	52
0	5	0
10,000		2,403

with the aim of obtaining a pattern of results showing what would happen just on the basis of chance. This approach can be used to quantify $\alpha[PF]$, the inflation of Type I error due to multiple testing. Equation 2 can then be used to set $\alpha[PC]$ in order to control the overall value of the Type I error.

As an illustration, suppose that 6 groups with 100 observations *per* group are created with data randomly sampled from a normal population. By construction, the H_0 is true (*i.e.,* all population means are equal). Now, construct 5 *independent contrasts* from these 6 groups. For each contrast, compute an F-test. If the probability associated with the statistical index is smaller than $\alpha = .05$, the contrast is said to reach significance (*i.e.,* $\alpha[PC]$ is used). Then have a computer redo the experiment $10,000$ times. In sum, there are $10,000$ experiments, $10,000$ families of contrasts and $5 \times 10,000 = 50,000$ contrasts. The results of this simulation are given in Table 1.

Table 1 shows that the H_0 is rejected for $2,403$ contrasts over the $50,000$ contrasts actually performed (5 contrasts times $10,000$ experiments). From these data, an estimation of $\alpha[PC]$ is computed as:

$$\begin{aligned}\alpha[PC] &= \frac{\text{number of contrasts having reached significance}}{\text{total number of contrasts}} \\ &= \frac{2,403}{50,000} = .0479 \ . \end{aligned} \tag{5}$$

This value falls close to the theoretical value of $\alpha = .05$.

It can be seen also that for $7,868$ experiments no contrast reached significance. Equivalently for $2,132$ experiments $(10,000 - 7,868)$ at least one Type I error was made. From these data, $\alpha[PF]$ can be estimated as:

$$\alpha[PF] = \frac{\text{number of families with at least 1 Type I error}}{\text{total number of families}} = \frac{2,132}{10,000} = .2132 \ . \qquad (6)$$

This value falls close to the theoretical value given by Equation 1:

$$\alpha[PF] = 1 - (1 - \alpha[PC])^C = 1 - (1 - .05)^5 = .226 \ .$$

2.2 Checking the orthogonality of two contrasts

Two contrasts are orthogonal (or independent) if their contrast coefficients are uncorrelated. Recall that contrast coefficients have zero sum (and therefore a zero mean). Therefore, two contrasts whose A contrast coefficients are denoted $C_{a,1}$ and $C_{a,2}$, will be orthogonal *if and only if*:

$$\sum_{a=1}^{A} C_{a,i} C_{a,j} = 0 \ . \qquad (7)$$

2.3 Computing sum of squares, mean square, and F

The sum of squares for a contrast can be computed using the C_a coefficients. Specifically, the sum of squares for a contrast is denoted SS_ψ, and is computed as

$$SS_\psi = \frac{S(\sum C_a M_{a.})^2}{\sum C_a^2} \qquad (8)$$

where S is the number of subjects in a group.

Also, because the sum of squares for a contrast has one degree of freedom it is equal to the mean square of effect for this contrast:

$$MS_\psi = \frac{SS_\psi}{df_\psi} = \frac{SS_\psi}{1} = SS_\psi \ . \qquad (9)$$

The F_ψ ratio for a contrast is now computed as

$$F_\psi = \frac{MS_\psi}{MS_{\text{error}}} \ . \qquad (10)$$

2.4 Evaluating F for orthogonal contrasts

Planned orthogonal contrasts are equivalent to independent questions asked to the data. Because of that independence, the current procedure is to act as if each contrast were the only contrast tested. This amounts to *not* using a correction for multiple tests. This procedure gives maximum power to the test. Practically, the null hypothesis for a contrast is tested by computing an F ratio as indicated in Equation 10 and evaluating its p value using a Fisher sampling distribution with $\nu_1 = 1$ and ν_2 being the number of degrees of freedom of MS_{error} [*e.g.,* in independent measurement designs with A groups and S observations per group $\nu_2 = A(S-1)$].

2.5 An example

This example is inspired by an experiment by Smith (1979). The main purpose in this experiment was to show that being in the same mental context for learning and for test gives better performance than being in different contexts. During the learning phase, subjects learned a list of 80 words in a room painted with an orange color, decorated with posters, paintings and a decent amount of paraphernalia. A first memory test was performed to give subjects the impression that the experiment was over. One day later, subjects were unexpectedly re-tested for their memory. An experimenter asked them to write down all the words of the list they could remember. The test took place in 5 different experimental conditions. Fifty subjects (ten *per* group) were randomly assigned to one of the five experimental groups. The five experimental conditions were:

1. *Same context.* Subjects are tested in the same room in which they learned the list.
2. *Different context.* Subjects are tested in a room very different from the one in which they learned the list. The new room is located in a different part of the campus, is painted grey and looks very austere.
3. *Imaginary context.* Subjects are tested in the same room as subjects from Group 2. In addition, they are told to try to remember the room in which they learned the list. In order to help them, the experimenter asks them several questions about the room and the objects in it.
4. *Photographed context.* Subjects are placed in the same condition as Group 3, and, in addition, they are shown photos of the orange room in which they learned the list.
5. *Placebo context.* Subjects are in the same condition as subjects in Group 2. In addition, before starting to try to recall the words, they are asked first to perform a warm-up task, namely, to try to remember their living room.

The data and ANOVA results of the replication of Smith's experiment are given in the Tables 2 and 3.

2.5.1 Research hypotheses for contrast analysis

Several research hypotheses can be tested with Smith's experiment. Suppose that the experiment was designed to test these hypotheses:

Table 2: Data from a replication of an experiment by Smith (1979). The dependent variable is the number of words recalled.

	Experimental Context				
	Group 1 Same	Group 2 Different	Group 3 Imagery	Group 4 Photo	Group 5 Placebo
	25	11	14	25	8
	26	21	15	15	20
	17	9	29	23	10
	15	6	10	21	7
	14	7	12	18	15
	17	14	22	24	7
	14	12	14	14	1
	20	4	20	27	17
	11	7	22	12	11
	21	19	12	11	4
$Y_{a.}$	180	110	170	190	100
$M_{a.}$	18	11	17	19	10
$M_{a.} - M_{..}$	3	−4	2	4	−5
$\sum(Y_{as} - M_{a.})^2$	218	284	324	300	314

Table 3: ANOVA table for a replication of Smith's experiment (1979).

Source	*df*	*SS*	*MS*	*F*	Pr(*F*)
Experimental	4	700.00	175.00	5.469**	.00119
Error	45	1,440.00	32.00		
Total	49	2,140.00			

- *Research Hypothesis 1.* Groups for which the context at test matches the context during learning (*i.e.,* is the same or is simulated by imaging or photography) will perform better than groups with a different or placebo contexts.
- *Research Hypothesis 2.* The group with the same context will differ from the group with imaginary or photographed contexts.
- *Research Hypothesis 3.* The imaginary context group differs from the photographed context group.
- *Research Hypothesis 4.* The different context group differs from the placebo group.

2.5.2 Contrasts

The four research hypotheses are easily transformed into statistical hypotheses. For example, the first research hypothesis is equivalent to stating the following null hypothesis:

> The means of the population for groups **1.**, **3.**, and **4.** have the same value as the means of the population for groups **2.**, and **5.**.

Table 4: Orthogonal contrasts for the replication of Smith (1979).

contrast	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	$\sum C_a$
ψ_1	+2	−3	+2	+2	−3	0
ψ_2	+2	0	−1	−1	0	0
ψ_3	0	0	+1	−1	0	0
ψ_4	0	+1	0	0	−1	0

This is equivalent to contrasting groups **1.**, **3.**, **4.** and groups **2.**, **5.**. This first contrast is denoted ψ_1:

$$\psi_1 = 2\mu_1 - 3\mu_2 + 2\mu_3 + 2\mu_4 - 3\mu_5 \ .$$

The null hypothesis to be tested is

$$H_{0,1}: \ \psi_1 = 0$$

The first contrast is equivalent to defining the following set of coefficients C_a:

Gr.1	Gr.2	Gr.3	Gr.4	Gr.5	$\sum_a C_a$
+ 2	− 3	+ 2	+ 2	− 3	0

Note that the sum of the coefficients C_a is zero, as it should be for a contrast. Table 4 shows all 4 contrasts.

2.5.3 Are the contrast orthogonal?

Now the problem is to decide if the contrasts constitute an orthogonal family. We check that every pair of contrasts is orthogonal by using Equation 7. For example, Contrasts 1 and 2 are orthogonal because

$$\sum_{a=1}^{A=5} C_{a,1}C_{a,2} = (2 \times 2) + (-3 \times 0) + (2 \times -1) + (2 \times -1) + (-3 \times 0) + (0 \times 0) = 0 \ .$$

2.5.4 F test

The sum of squares and F_ψ for a contrast are computed from Equations 8 and 10. For example, the steps for the computations of $SS_{\psi 1}$ are given in Table 5:

Table 5: Steps for the computation of $SS_{\psi 1}$ of Smith (1979).

Group	M_a	C_a	$C_a M_a$	C_a^2
1	18.00	+2	+36.00	4
2	11.00	−3	−33.00	9
3	17.00	+2	+34.00	4
4	19.00	+2	+38.00	4
5	10.00	−3	−30.00	9
		0	45.00	30

$$
\begin{aligned}
SS_{\psi 1} &= \frac{S(\sum C_a M_{a.})^2}{\sum C_a^2} = \frac{10 \times (45.00)^2}{30} = 675.00 \\
MS_{\psi 1} &= 675.00 \\
F_{\psi 1} &= \frac{MS_{\psi 1}}{MS_{\text{error}}} = \frac{675.00}{32.00} = 21.094 \ . \qquad (11)
\end{aligned}
$$

The significance of a contrast is evaluated with a Fisher distribution with 1 and $A(S-1) = 45$ degrees of freedom, which gives a critical value of 4.06 for $\alpha = .05$ (7.23 for $\alpha = .01$). The sum of squares for the remaining contrasts are $SS_{\psi.2} = 0$, $SS_{\psi.3} = 20$, and $SS_{\psi.4} = 5$ with 1 and $A(S-1) = 45$ degrees of freedom. Therefore, ψ_2, ψ_3, and ψ_4 are non-significant. Note that the sums of squares of the contrasts add up to $SS_{\text{experimental}}$. That is:

$$
\begin{aligned}
SS_{\text{experimental}} &= SS_{\psi.1} + SS_{\psi.2} + SS_{\psi.3} + SS_{\psi.4} \\
&= 675.00 + 0.00 + 20.00 + 5.00 \\
&= 700.00 \ .
\end{aligned}
$$

When the sums of squares are orthogonal, the degrees of freedom are added the same way as the sums of squares are. This explains why the maximum number of orthogonal contrasts is equal to number of degrees of freedom of the experimental sum of squares.

3 A priori (planned) non-orthogonal contrasts

So, orthogonal contrasts are relatively straightforward because each contrast can be evaluated on its own. Non-orthogonal contrasts, however, are more complex. The main problem is to assess the importance of a given contrast conjointly with the other contrasts. There are currently two (main) approaches to this problem. The classical approach corrects for multiple statistical tests (*e.g.,* using a Šidàk or Bonferroni correction), but essentially evaluates each contrast as if it were coming from a set of orthogonal contrasts. The multiple regression (or modern) approach evaluates each contrast as a predictor from a set of non-orthogonal predictors and estimates its *specific* contribution to the explanation of the dependent variable. The classical approach evaluates each contrast for itself, whereas the multiple regression approach evaluates each contrast as a member of a set of contrasts and estimates the specific contribution of each contrast in this set. For an orthogonal set of contrasts, the two approaches are equivalent.

3.1 The classical approach

Some problems are created by the use of multiple non-orthogonal contrasts. Recall that the most important one is that the greater the number of contrasts, the greater the risk of a Type I error. The general strategy adopted by the classical approach to take this problem is to correct for multiple testing.

3.1.1 Šidàk and Bonferroni corrections for non-orthogonal contrasts

When a family of contrasts are nonorthogonal, Equation 1 gives a lower bound for $\alpha[PC]$ (*cf.* Šidàk, 1967; Games, 1977). So, instead of having the equality, the following inequality, called the Šidàk inequality, holds

$$\alpha[PF] \leq 1 - (1 - \alpha[PC])^C \ . \quad (12)$$

This inequality gives an upper bound for $\alpha[PF]$, therefore the real value of $\alpha[PF]$ is smaller than its estimated value.

As previously, we can approximate the Šidàk inequality by Bonferroni as

$$\alpha[PF] < C\alpha[PC] \ . \quad (13)$$

And, as previously, Šidàk and Bonferroni are linked to each other by the inequality

$$\alpha[PF] \leq 1 - (1 - \alpha[PC])^C < C\alpha[PC] \ . \quad (14)$$

3.1.2 An example: Classical approach

Let us go back to Smith's (1979) study (see Table 2). Suppose that Smith wanted to test these three hypotheses:

– *Research Hypothesis 1.* Groups for which the context at test matches the context during learning will perform better than groups with different contexts;

Table 6: Non-orthogonal contrasts for the replication of Smith (1979).

contrast	Gr. 1	Gr. 2	Gr. 3	Gr. 4	Gr. 5	$\sum C_a$
ψ_1	2	−3	2	2	−3	0
ψ_2	3	3	−2	−2	−2	0
ψ_3	1	−4	1	1	1	0

Table 7: F_ψ values for the nonorthogonal contrasts from the replication of Smith (1979).

	$r_{Y.\psi}$	$r^2_{Y.\psi}$	F_ψ	$p(F_\psi)$
ψ_1	.9820	.9643	21.0937	< .0001
ψ_2	−.1091	.0119	0.2604	.6123
ψ_3	.5345	.2857	6.2500	.0161

– *Research Hypothesis 2.* Groups with real contexts will perform better than those with imagined contexts;
– *Research Hypothesis 3.* Groups with any context will perform better than those with no context.

These hypotheses can easily be transformed into the set of contrasts given in Table 6. The values of F_ψ were computed with Equation 10 (see also Table 3) and are in shown in Table 7 along with their p values. If we adopt a value of $\alpha[PF] = .05$, a Šidàk correction (from Equation 2) will entail evaluating each contrast at the α level of $\alpha[PC] = .0170$ (Bonferroni will give the approximate value of $\alpha[PC] = .0167$). So, with a correction for multiple comparisons we will conclude that Contrasts 1 and 3 are significant.

3.2 Multiple regression approach

Anova and multiple regression are equivalent if we use as many predictors for the multiple regression analysis as the number of degrees of freedom of the independent variable. An obvious choice for the predictors is to use a set of contrasts coefficients. Doing so makes contrast analysis a particular case of multiple regression analysis. When used with a set of orthogonal contrasts, the multiple regression approach gives the same results as the anova based approach previously described. When used with a set of *non-orthogonal* contrasts, multiple regression quantifies the *specific* contribution of each contrast as the semi-partial coefficient of correlation between the contrast coefficients and the dependent variable. We can use the multiple regression approach for non-orthogonal contrasts as long as the following constraints are satisfied:

1. There are no more contrasts than the number of degrees of freedom of the independent variable;
2. The set of contrasts is linearly independent (*i.e.,* not multicollinear). That is, no contrast can be obtained by combining the other contrasts.

3.2.1 An example: Multiple regression approach

Let us go back once again to Smith's (1979) study of learning and recall contexts. Suppose we take our three contrasts (see Table 6) and use them as predictors with a standard multiple regression program. We will find the following values for the semi-partial correlation between the contrasts and the dependent variable:

$$\begin{aligned}\psi_1: &\quad r^2_{Y.C_{a,1}|C_{a,2}C_{a,3}} = .1994\\ \psi_2: &\quad r^2_{Y.C_{a,2}|C_{a,1}C_{a,3}} = .0000\\ \psi_3: &\quad r^2_{Y.C_{a,3}|C_{a,1}C_{a,2}} = .0013\ ,\end{aligned}$$

with $r^2_{Y.C_{a,1}|C_{a,2}C_{a,3}}$ being the squared correlation of ψ_1 and the dependent variable with the effects of ψ_2 and ψ_3 partialled out. To evaluate the significance of each contrast, we compute an F ratio for the corresponding semi-partial coefficients of correlation. This is done using the following formula:

$$F_{Y.C_{a,i}|C_{a,k}C_{a,\ell}} = \frac{r^2_{Y.C_{a,i}|C_{a,k}C_{a,\ell}}}{1 - r^2_{Y.A}} \times (df_{\text{residual}})\ . \tag{15}$$

This results in the following F ratios for the Smith example:

$$\begin{aligned}\psi_1: &\quad F_{Y.C_{a,1}|C_{a,2}C_{a,3}} = 13.3333, \quad p = 0.0007;\\ \psi_2: &\quad F_{Y.C_{a,2}|C_{a,1}C_{a,3}} = 0.0000, \quad p = 1.0000;\\ \psi_3: &\quad F_{Y.C_{a,3}|C_{a,1}C_{a,2}} = 0.0893, \quad p = 0.7665.\end{aligned}$$

These F ratios follow a Fisher distribution with $\nu_1 = 1$ and $\nu_2 = 45$ degrees of freedom. $F_{\text{critical}} = 4.06$ when $\alpha = .05$. In this case, ψ_1 is the only contrast reaching significance (*i.e.*, with $F_\psi > F_{\text{critical}}$). The comparison with the classic approach shows the drastic differences between the two approaches.

4 A posteriori (post-hoc) contrasts

For *a posteriori* contrasts, the family of contrasts is composed of all the possible contrasts even if they are not explicitly made. Indeed, because we *choose* the contrasts to be made *a posteriori*, this implies that we have *implicitly* made and judged uninteresting *all* the possible contrasts that have not been made. Hence, whatever the number of contrasts actually performed, the family is composed of all the possible contrasts. This number grows very fast: A conservative estimate indicates that the number of contrasts which can be made on A groups is equal to

$$1 + \{[(3^A - 1)/2] - 2^A\}\ . \tag{16}$$

So, using a Šidàk or Bonferroni approach will not have enough power to be useful.

4.1 Scheffé's test

Scheffé's test was devised to test all possible contrasts *a posteriori* while maintaining the overall Type I error level for the family at a reasonable level, as well as trying to have a conservative but relatively powerful test. The general principle is to insure that no discrepant statistical decision can occur. A discrepant decision would occur if the omnibus test would fail to reject the null hypothesis, but one *a posteriori* contrast could be declared significant.

In order to avoid such a discrepant decision, the Scheffé approach first tests any contrast as if it were the largest possible contrast whose sum of squares is equal to the experimental sum of squares (this contrast is obtained when the contrast coefficients are equal to the deviations of the group means to their grand mean); and second makes the test of the largest contrast equivalent to the ANOVA omnibus test. So, if we denote by $F_{\text{critical,omnibus}}$ the critical value for the ANOVA omnibus test (performed on A groups), the largest contrast is equivalent to the omnibus test if its F_ψ is tested against a critical value equal to

$$F_{\text{critical, Scheffé}} = (A-1) \times F_{\text{critical,omnibus}}. \tag{17}$$

Equivalently, F_ψ can be divided by $(A-1)$ and its probability can be evaluated with a Fisher distribution with $\nu_1 = (A-1)$ and ν_2 being equal to the number of degrees of freedom of the mean square error. Doing so makes it *impossible* to reach a discrepant decision.

4.1.1 An example: Scheffé

Suppose that the F_ψ ratios for the contrasts computed in Table 7 were obtained *a posteriori.* The critical value for the ANOVA is obtained from a Fisher distribution with $\nu_1 = A-1 = 4$ and $\nu_2 = A(S-1) = 45$. For $\alpha = .05$ this value is equal to $F_{\text{critical,omnibus}} = 2.58$. In order to evaluate if any of these contrasts reaches significance, we need to compare them to the critical value of

$$F_{\text{critical,Scheffé}} = (A-1) \times F_{\text{critical,omnibus}} = 4 \times 2.58 = 10.32.$$

With this approach, only the first contrast is considered significant.

Related entries

Analysis of Variance, Bonferonni correction, Post-Hoc comparisons.

Further Readings

1. Abdi, H., Edelman, B., Valentin, D., & Dowling, W.J. (2009). *Experimental Design and Analysis for Psychology.* Oxford: Oxford University Press.
2. Rosenthal, R., & Rosnow, R.L. (2003). *Contrasts and effect sizes in behavioral research: A correlational approach.* Boston: Cambridge University Press.