Digital Phase Detecting Based On Hilbert Transform and Its DSP Implementation

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Abstract In direction finding (DF) with phase interferometer, it is very important to accurately detect the phase difference between two coherent signals for DF system. Based on the 90° phase-shift characteristic of Hilbert transform, a digital phase detecting method is introduced in this paper. The phase difference of two coherent signals can be measured by Hilbert transform and simple mathematical computation. FFT and IFFT calculations, instead of complicated integral, are taken to realize the digital Hilbert transformation. As a result, the computing speed is raised a lot. The results of software and hardware simulation show that, this method has many merits, such as high accuracy, great speed and its good performance for low SNR signals in especial, which make it easy to realize. Finally, it is put into practical use by the development of DF processing board with DSP (Digital Signal Processor) and PCI bus.

Key words: phase interferometer; direction finding; Hilbert transform; digital phase detecting; FFT

INTRODUCTION

In general, Non-source Direction Finding (DF) systems[1] can be classified as two categories: in one way, direction is found from the signal's changes by receiving antenna's beamforming movement; in another, comparison of several received signals from multi antennae is utilized. The research in this paper is based on the latter with single receiver. In this system, there are four DF methods such as amplitude comparing, phase comparing, time difference comparing[2] and complete waveform comparing. The second one, i.e. phase interfering, is adopted here.

Design of phase-detector is the key point for the realization of phase interfering method. Although domestic techniques of analog phase detector have developed well till now, yet its precision is not stable due to the poor thermal characteristics of analog components. Therefore, digital design is going to be the tendence.

Albeit there are many digital phase detecting methods, most of them are not satisfactory with regard to precision or real time performance.. In this paper based on Hilbert transform[3], digital phase detecting is realized with high accuracy. In addition, influence by additive noises is reduced by mathematical expectation, which lower the requirements for received signals.

1. PHASE INTERFEREOMETER FOR DF

Given a two-antenna array, time delay between two antennae is

$$\tau = \frac{d\sin\theta}{c} \qquad (1)$$

where d , θ and c represent the distance between two antennae, the arriving azimuth and the velocity of light, respectively.

Two received signals from antenae can be mathematically expressed as

$$s_a(t) = a\cos(\omega t + \phi)$$

$$s_b(t) = a\cos(\omega(t-\tau)+\phi)$$

From（1）, it can be obtained that the phase difference is

$$\Delta\phi = \omega\tau = \omega\frac{d\sin\theta}{c} \qquad (2)$$

which can be rewritten to obtain the arriving azimuth

$$\theta = \arcsin(\Delta\phi\frac{c}{\omega d}) \quad . \qquad (3)$$

Therefore, phase difference of received signals from two antennae, denoted by $\Delta\phi$, should be measured first in order to find direction.

2. PRINCIPLE

2．1 Realization of 90° phase shift with Hilbert transform

Hilbert transform, an important mathematical tool for analytical signals, can be expressed in temporal domain as

$$\hat{s}(t) = H[s(t)] = s(t) * \frac{1}{\pi t}$$
$$= \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{s(t-\tau)}{\tau}d\tau \qquad (4)$$

which is equivalent to 90° phase shift linear filter, as shown in Fig.1.

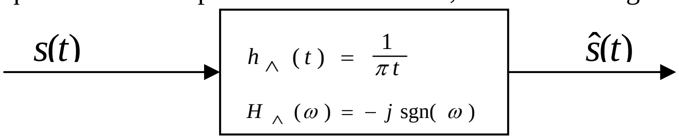

Fig. 1 Equivalent 90° phase shift linear filter for Hilbert transform

Application of (4) to a restricted band signal $s(t) = a(t)\cos(\omega_0 t + \phi)$ leads to its Hilbert transformation that

$$\hat{s}(t) = H[s(t)] = a(t)\sin(\omega_0 t + \phi) \qquad (5)$$

2．2 Digital phase detection

Assuming received signals in two channels are

$$s_1(t) = a_1(t)\cos(\omega t + \phi_1) \qquad (6)$$

$$s_2(t) = a_2(t)\cos(\omega t + \phi_2), \qquad (7)$$

the function of phase detector is to measure the phase difference between two coherent signals, i.e. $\Delta\phi = \phi_2 - \phi_1$.

From (5), Hilbert transforms of two signals are

$$H[s_1(t)] = a_1(t)\sin(\omega t + \phi_1) \qquad (8)$$

$$H[s_2(t)] = a_2(t)\sin(\omega t + \phi_2) \quad . \qquad (9)$$

Denote two indirect variable I, Q that

$$I = s_1(t)\cdot H[s_2(t)] - H[s_1(t)]\cdot s_2(t)$$

$$= a_1(t)\cos(\omega t+\phi_1)\cdot a_2(t)\sin(\omega t+\phi_2) - a_1(t)\sin(\omega t+\phi_1)\cdot a_2(t)\cos(\omega t+\phi_2)$$

$$= a_1(t)\cdot a_2(t)\cdot \sin(\phi_2-\phi_1) \qquad (10)$$

$$Q = s_1(t)\cdot s_2(t) + H[s_1(t)]\cdot H[s_2(t)]$$

$$= a_1(t)\cos(\omega t+\phi_1)\cdot a_2(t)\cos(\omega t+\phi_2) + a_1(t)\sin(\omega t+\phi_1)\cdot a_2(t)\sin(\omega t+\phi_2)$$

$$= a_1(t)\cdot a_2(t)\cdot \cos(\phi_2-\phi_1), \qquad （11）$$

divide (10) by（11）and it yields

$$I/Q = \frac{\sin(\phi_2-\phi_1)}{\cos(\phi_2-\phi_1)} = \tan(\phi_2-\phi_1), \qquad (12)$$

from which phase difference, $\Delta\phi = \phi_2 - \phi_1$, of two coherent signals can be obtained by arc tangent function.

When implemented with DSP, only simple mathematic operations, such as multiplication, addition and subtration are needed to lead to the value of I，Q, which can yield the phase difference. In addition, the values of I，Q at each sampling are not constants due to additive noise. Thereby statistic method is adopted.

2．3 Realization of digital Hilbert transform

It is considerably complicated to realize digital Hilbert transform using integral in temporal domain directly. More simply, it is fulfilled in frequency domain instead, which give rise to the computing speed.

According to Hilbert transform properties, the spectrum of analytical signal is double the value of the initial one in positive frequency domain, whereas zero in negative frequency domain. Hence, we implement Hilbert filter[4] using indirect transform. Initial signal is processed with FFT transform firstly, with negative frequency domain cleared to zero. Then IFFT transform is used to yield the result, of which the imaginary part is the expected Hilbert transform of the initial signal. The flow chart is shown in Fig.2.

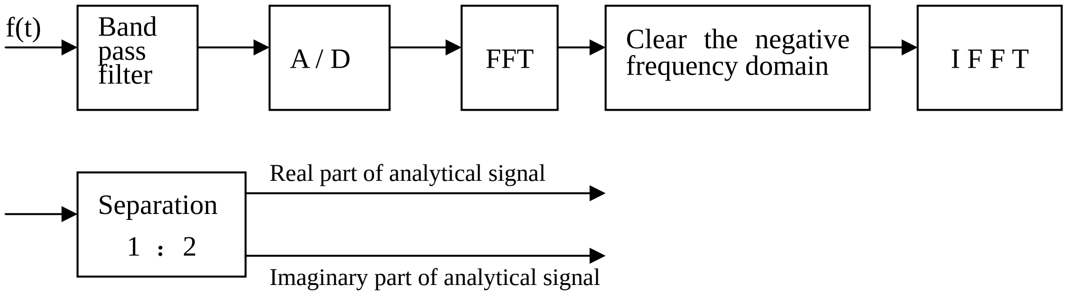

Fig.2　Flow chart of digital Hilbert transform

3. SIMULATION

3．1 Software simulation

With aid of Matlab5.3, simulation is conducted to FM signal, with different effects shown when signal aberrance, signal to noise(S/N) ratio and sampling quantity vary.

Two channels of received signals is mathematically modeled as

$$s_1 = (1 + A\cos 2\pi f_n t)\cdot\cos(2\pi f_c t + k\sin 2\pi f_0 t) + n_1(t)$$

$$s_2 = (1 + A\cos 2\pi f_n t)\cdot\cos(2\pi f_c t + k\sin 2\pi f_0 t + \Delta\phi) + n_2(t)\,,$$

where f_n =1kHz, f_c =455kHz, f_0 =10kHz, k=11.5, SNR=10dB, $\Delta\phi = 45^\circ$ and $n_1(t)$, $n_2(t)$ represent random noises in two channels, respectively.

3．1．1 Comparison of different signal aberrance

Notice that parasitic amplitude modulation of FM signal will cause aberrance, which will debase the precision of usual analog phase detector.

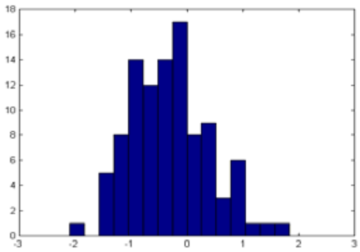

Fig.3 Error distributing histogram of non-aberrance signals (SNR=10dB, sampling points=1024)

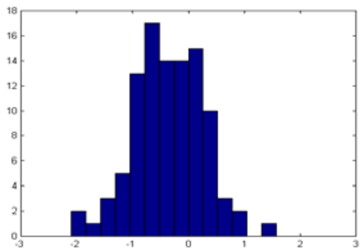

Fig.4 Error distributing histogram of aberrant signals (SNR=10dB,sampling points=1024)

100 trials are conducted to signals with and without aberrance, with error distributing histogram shown in Fig.3 and Fig.4. Statistic errors 0.8603° and 0.8651°(RMS) indicate that aberrance has almost no influence on its performance.

3．1．2 Simulation in low SNR situation

Owing to complex environment, signals are always seriously polluted by noises in practical use. Therefore, it is necessary to test its performance in low SNR situation in order to ensure its function.

Simulation in 8dB situation is shown in Fig.5 and the RMS error is 0.9638°. Comparing with the 10dB situation shown in Fig.3, it leads us to believe that precision will not be influenced much under low S/N ratio conditions.

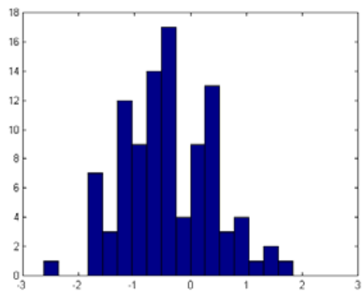

Fig.5 Error distributing histogram when SNR=8dB (non-aberrance，sampling points=1024)

3. 1. 3 Simulation when sampling points are increased

Error distributing histogram of 100 trials when sampling points are increased to 4096 is shown in Fig.6, compared with Fig.3 when sampling 1024 points.

RMS error is 0.3936°. It indicates that higher precision is achieved. However, more time is needed on the other hand. Hence, compromise between precision and speed must be considered in practice.

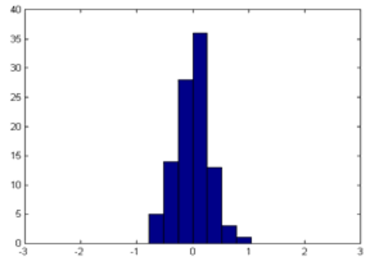

Fig.6 Error distributing histogram when sampling 4096 points (non-aberrance,SNR=10dB)

3．2 Hardware simulation

Hardware simulation system: TMS320VC5410 EVM board with DSP of 100 MIPS speed , 64kB on-chip RAM and 16kB on-chip ROM.

Developing environment: Code Composer Studio 1.20[5] using C54x assembly language[6].

Hardware simulation is conducted in situation of 45° phase difference, 10dB SNR, 10MHz sampling rate and 1024 sampling points. Comparison between software and hardware simulation results is shown in Table 1.

Table 1. Comparison between software and hardware simulation results

Software simulation results (deg)	45.7617	44.7150	45.5692	45.8940	45.9491	45.9108	45.3699	46.4346	45.2628	45.6226
Hardware simulation results(deg)	45.2421	45.4960	45.3995	44.8501	44.7468	45.1503	45.3043	46.6315	45.1922	45.5103

It cost 2.2ms for DSP running, which can be reduced to 1.375ms if 160MIPS DSP is used. Comparison shows that effect of hardware simulation is also desirable.

4. SYSTEM IMPLEMENTATION

This system is PCI bus based digital processor, that consists of PCI2040 bridge, TMSVC5410DSP[7], 12-Bit, 25MSPS A/D converter and Complex Programmable Logic Device (CPLD).

Before entering this system, signals in two channels from antennae have been processed by mixing circuit, which changed the high frequency signal into intermediate frequency of 455kHz. During system's running, sampled signal is converted into digital one by A/D converter. DSP processes these data with the method introduced above, and then transmits the result to PC through PCI bus for later use. System frame is depicted in Fig.7.

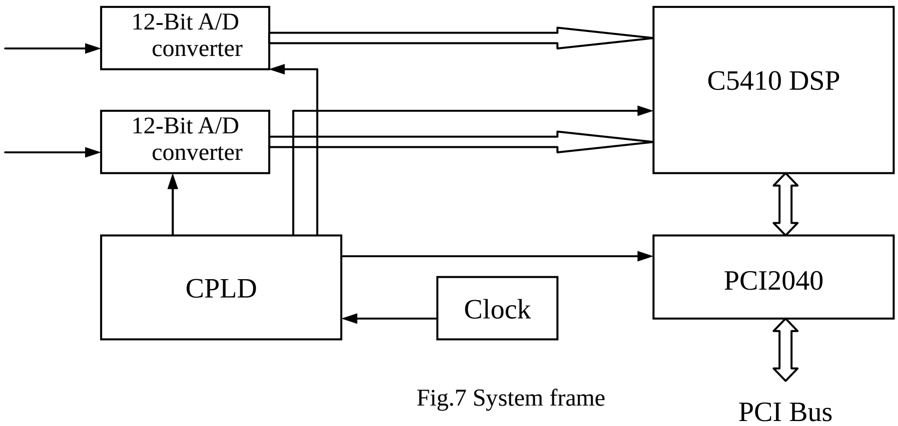

Fig.7 System frame

5. CONCLUSIONS

This paper has addressed a digital phase detecting method making use of 90° phase shift property of Hilbert transform. Phase difference of two intermediate frequency signals can be easily obtained by simple calculations and FFT, IFFT transform. Result of software and hardware simulation show that this system meets the precision and real time requirements under aberrance and low SNR conditions. Although in this paper FM signal is mainly analyzed, yet it can be applied to QPSK signal of CDMA system and GMSK signal of GSM system as well. Finally, PCI bus based DSP digital phase detector for 455kHz signal has been successful developed, substituting the initial analog device in a Radio Supervisory System.

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