RESEARCH ON NONLINEAR DISTORTION IN AMPLIFIERS

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ABSTRACT

Transistors are nonlinear devices, which can produce nonlinear distortion in amplifier while amplifying signals. For weak nonlinear distortion, the expressions of total harmonic distortion (THD), the second-order intermodulation distortion ($IM2$), the third-order intermodulation distortion (IM_3) and intercept point (IP_3) are deduced. With the aid of software Multisim, we simulate transistor common-emitter amplifier, transistor common-emitter amplifier with resistor in emitter, differential amplifier and differential amplifier with resistor between emitters. The simulational results and theoretical analyses are almost the same.

Key words: nonlinear distortion; intermodulation distortion; intercept point; negative feedback; differential amplifier

INTRODUCTION

The input characteristic $i_B = f(u_{BE})|_{u_{CE}=\text{constant}}$ and the output characteristic $i_C = f(u_{CE})|_{i_B=\text{constant}}$ of transistor are both nonlinear. Therefore, an appropriate operating point (usually denoted by Q) is necessary for transistors operating within the approximately linear range, so that the input signal can be amplified without distortion (or distortion is within the permissive range). It is very important to select the operating point, because neither can it be too close to the saturation region, nor to the cut-off region. Otherwise, nonlinear distortion will be generated, when the input signal is large enough to cause the transistor to work in the saturation region or in the cut-off region.

Distortion of the amplifier can be classified as linear and nonlinear ones. Due to the influence of coupling capacitance, junction capacitance and by-pass capacitance, a frequency band exists for the circuit. Within the passing band, the amplifying times and the phase shift are almost the same. Out of the passing band, the amplifying times will decrease and the change of phase shift will take place (increase or decrease). When a certain signal is the input of the amplifier, such as a square waveform composed of different frequency components, the amplifying times and phase shifts vary with the different frequency components. So the output signal will be distorted, which is called linear distortion. The most obvious characteristic of linear distortion is that no new frequency component is generated in the output signal.

Nonlinear distortion is caused by a nonlinear transfer characteristic. The waveform of the output is different from that of the input. This corresponds to the generation of new frequency components, which is the characteristic of this kind of distortion.

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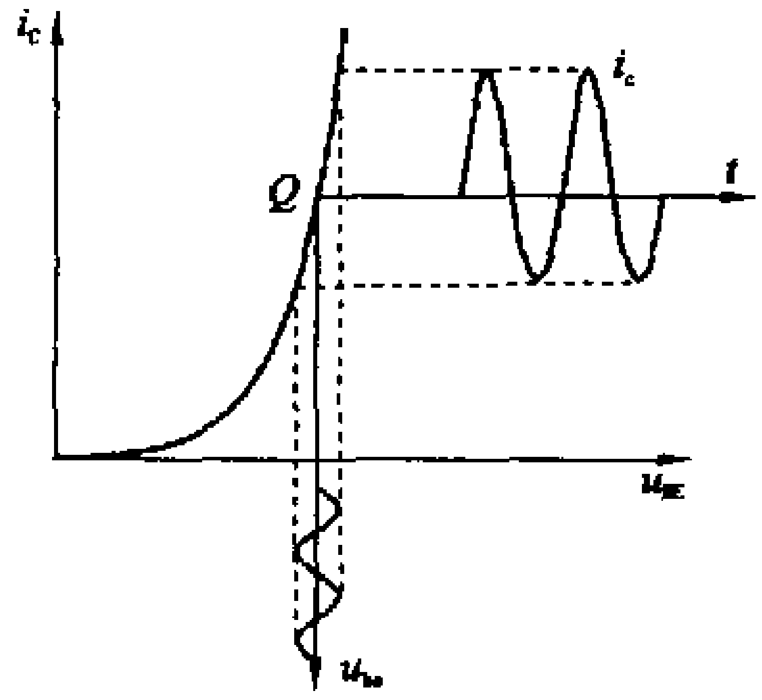
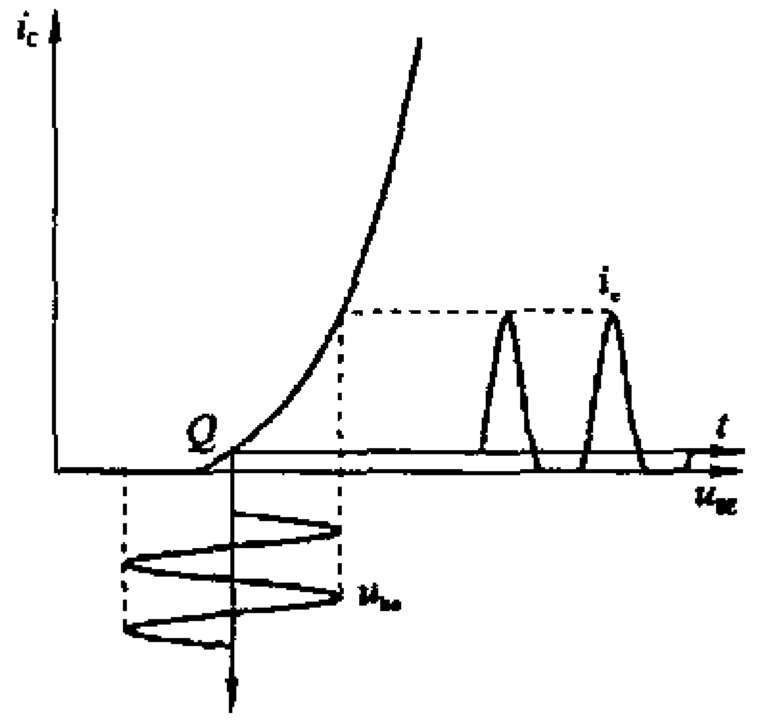
According to the degree, nonlinear distortion is classified as weak and hard ones, as shown in Fig. 1(a,b).



(a) Weak nonlinear distortion



(b) Hard nonlinear distortion

Fig. 1 Nonlinear distortion

Since hard nonlinear distortion can be avoided by adjusting the operating point or reducing the input signal amplitude, they will not be discussed any more in this paper. Then let us consider the characteristic and the improving measures of weak nonlinear distortion, as shown in Fig. 1(a).

1 WEAK NONLINEAR DISTORTION AND ITS INDEX

Let us consider an amplifier with a weak nonlinear distortion as in Fig. 1(a). The output collector ac current $i_c(t)$, simplified by i_c, can be expressed in terms of the input emitter ac voltage by a power series

$$i_c = a_0 + a_1 u_{be} + a u_{be}^2 + a_3 u_{be}^3 + \cdots \quad (1)$$

where

$$a_n = \frac{1}{n!}\frac{d^n i_c}{d^n u_{BE}}\Bigg|_{u_{BE}=U_{BEQ}} \quad (2)$$

Coefficient a_0 represents the dc component of the output signal. a_1 represents the linear gain of the amplifier and $a_2, a_3, \cdots, a_n$ represent coefficients of distortion terms.

Under the condition of weak nonlinear distortion, the terms whose order is higher than 3 can be neglected and Eq. (1) can be simplified as

$$i_c = a_0 + a_1 u_{be} + a_2 u_{be}^2 + a_3 u_{be}^3 \quad (3)$$

Application of a cosine waveform $u_{be} = U_m \cos\omega t$ as input signal to Eq. (3) yields

$$i_c = a_0 + \frac{1}{2} a_2 U_m^2 \qquad \text{DC component}$$

$$+ (a_1 + \frac{3}{4} a_3 U_m^2) U_m \cos\omega t$$

$$\text{Fundamental component}$$

$$+ \frac{1}{2} a_2 U_m^2 \cos 2\omega t \quad \text{Second-order component}$$

$$+ \frac{1}{4} a_3 U_m^3 \cos 3\omega t$$

$$\text{Third-order component} \quad (4)$$

Harmonic distortion is usually used to represent the amplifier distortion. The nth harmonic distortion, with second-order, third-order harmonic distortion considered mainly, is then defined as the ratio of the component of frequency $n\omega$ to the one at the fundamental ω. Application to Eq. (4) yields the second-order harmonic distortion coefficient HD_2 as

$$HD_2 = \frac{1}{2} \frac{a_2}{a_1 + \frac{3}{4} a_3 U_m^2} U_m \approx \frac{1}{2} \frac{a_2}{a_1} U_m \quad (5)$$

and the third-order harmonic distortion coefficient HD_3 as

$$HD_3 = \frac{1}{4} \frac{a_3}{a_1 + \frac{3}{4} a_3 U_m^2} U_m^2 \approx \frac{1}{4} \frac{a_3}{a_1} U_m^2 \quad (6)$$

It can be seen that HD_2 is approximately proportional to U_m and HD_3 to U_m^2.

The total harmonic distortion coefficient THD is defined as

$$THD = \sqrt{HD_2^2 + HD_3^2 + \cdots} \tag{7}$$

Also the amplifier distortion can be measured by the intermodulation distortion (IM). Use of an input signal, such as $u_{be} = U_m\cos\omega_1 t + U_m\cos\omega_2 t$ (let $\omega_1 > \omega_2$), which is composed of two frequencies close to each other yields

$$\begin{aligned} i_c =& a_0 + a_1(U_m\cos\omega_1 t + U_m\cos\omega_2 t) + \\ & a_2(U_m\cos\omega_1 t + U_m\cos\omega_2 t)^2 + \\ & a_3(U_m\cos\omega_1 t + U_m\cos\omega_2 t)^3 = \\ & a_0 + a_2 U_m^2 + a_2 U_m^2\cos(\omega_2 - \omega_1)t + \\ & a_2 U_m^2\cos(\omega_2 + \omega_1)t + \\ & (a_1 U_m + \frac{9}{4}a_3 U_m^3)\cos\omega_1 t + \\ & (a_1 U_m + \frac{9}{4}a_3 U_m^3)\cos\omega_2 t + \\ & \frac{3}{4}a_3 U_m^3\cos(2\omega_1 - \omega_2)t + \\ & \frac{3}{4}a_3 U_m^3\cos(2\omega_2 - \omega_1)t + \\ & \frac{3}{4}a_3 U_m^3\cos(2\omega_1 + \omega_2)t + \\ & \frac{3}{4}a_3 U_m^3\cos(2\omega_2 + \omega_1)t + \\ & \frac{1}{2}a_2 U_m^2\cos 2\omega_1 t + \frac{1}{2}a_2 U_m^2\cos 2\omega_2 t + \\ & \frac{1}{4}a_3 U_m^3\cos 3\omega_1 t + \frac{1}{4}a_3 U_m^3\cos 3\omega_2 t \end{aligned} \tag{8}$$

Second-order intermodulation distortion (IM_2) is defined as the ratio of the component at frequencies $\omega_1 \pm \omega_2$ to the one at ω_1 or ω_2. Application to Eq. (8) yields

$$IM_2 \approx \frac{a_2}{a_1}U_m \tag{9}$$

Third-order intermodulation distortion (IM_3) is defined as the ratio of the component at frequencies $2\omega_2 \pm \omega_1, 2\omega_1 \pm \omega_2$ to the one at ω_1, ω_2. Application to Eq. (8) yields

$$IM_3 \approx \frac{3}{4}\frac{a_3}{a_1}U_m^2 \tag{10}$$

Comparison of Eqs. (9, 10) with Eqs. (5, 6) yields

$$IM_2 = 2HD_2 \tag{11}$$

$$IM_3 = 3HD_3 \tag{12}$$

The relation curves of the fundamental component $a_1 U_m$, third-order intermodulation component $\frac{3}{4}a_3 U_m^2$ versus the amplitude of input signal are shown in Fig. 2, with input signal amplitude U_m as horizontal axis and output signal amplitude as vertical axis.

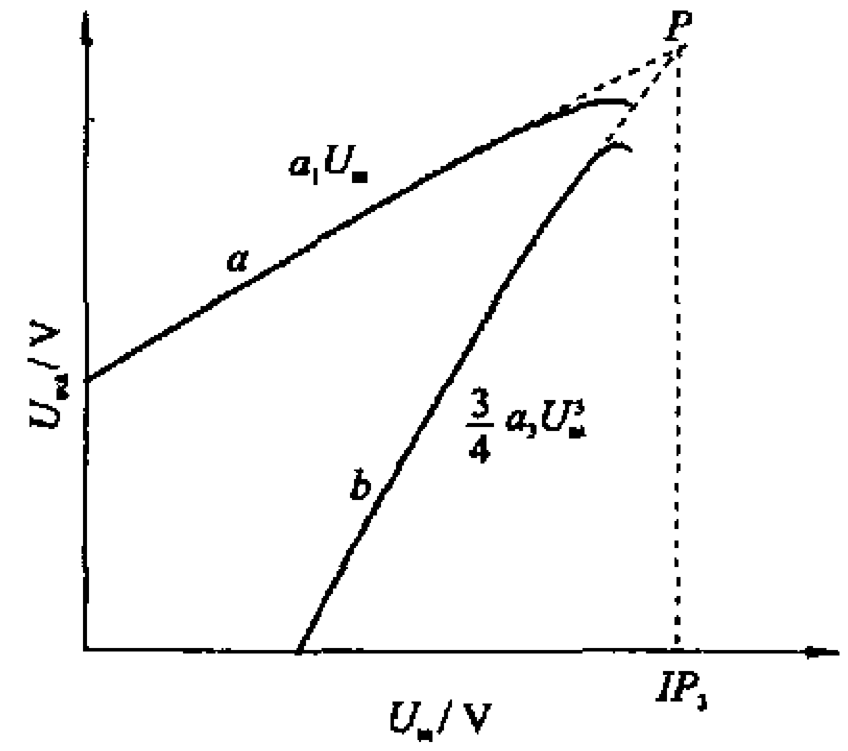


Fig. 2 Fundamental and IM3 component versus input voltage

The point where the extrapolated curves of a and b coincide is called third-order intercept point denoted by P. Application of the characteristic of P yields

$$IP_3 = \sqrt{\frac{4}{3}\frac{a_1}{a_3}} \tag{13}$$

Note that IP_3 represents the input signal amplitude when $IM_3 = 1$. The larger the IP_3, the better the linear characteristic of the amplifier and the smaller the distortion.

2 DISTORTION ANALYSES OF COMMON EMITTER AMPLIFIER AND DIFFERENTIAL AMPLIFIER

The collector current of the common emitter amplifier, shown in Fig. 3(a) is given by

$$i_C = I_S e^{\frac{u_{BE}}{U_T}} \tag{14}$$

which can be expressed by the sum of the dc component I_{CQ} and the ac component i_c, i. e. , $i_C =$

$I_{CQ}+i_c$. Emitter voltage u_{BE} can be expressed as $u_{BE}=U_{BE}+u_{be}$. Hence, Eq. (14) results in

$$i_c = I_{CQ}(e^{\frac{u_{be}}{U_T}} - 1) \quad (15)$$

For small input signals, i.e., $u_{be} \ll U_T$, Eq. (15) can be expanded in a Taylor series with high order terms neglected as

$$i_c = I_{CQ}\left[\frac{u_{be}}{U_T} + \frac{1}{2!}\left(\frac{u_{be}}{U_T}\right)^2 + \frac{1}{3!}\left(\frac{u_{be}}{U_T}\right)^3\right] \quad (16)$$

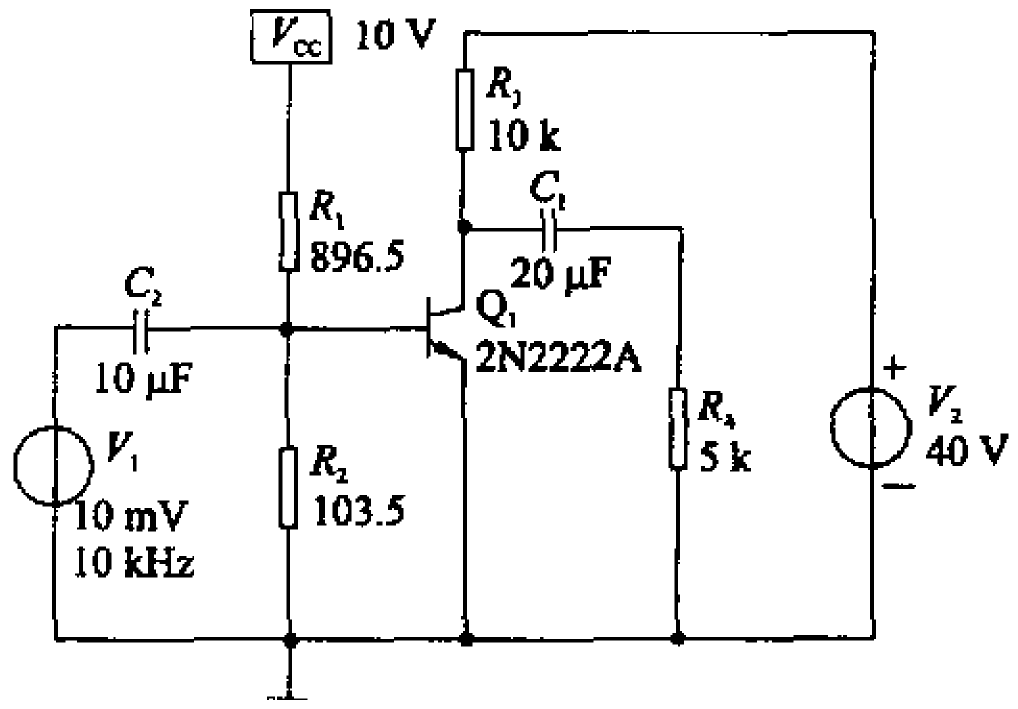
Use of Eqs. (5, 6, 9, 10, 13) and substitution of $u_{be}=U_m\cos\omega t$ yields

$$IM_2 = 2HD_2 = \frac{1}{2}\frac{U_m}{U_T} \quad (17)$$

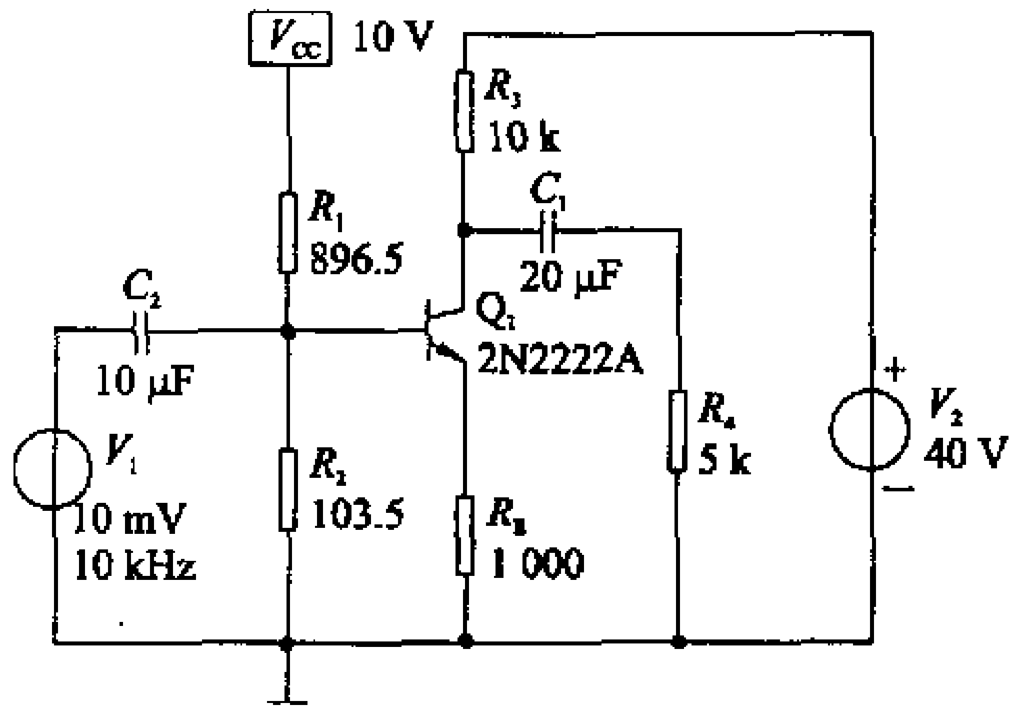
$$IM_3 = 3HD_3 = \frac{1}{8}\left(\frac{U_m}{U_T}\right)^2 \quad (18)$$

$$IP_3 = \sqrt{8}U_T \quad (19)$$

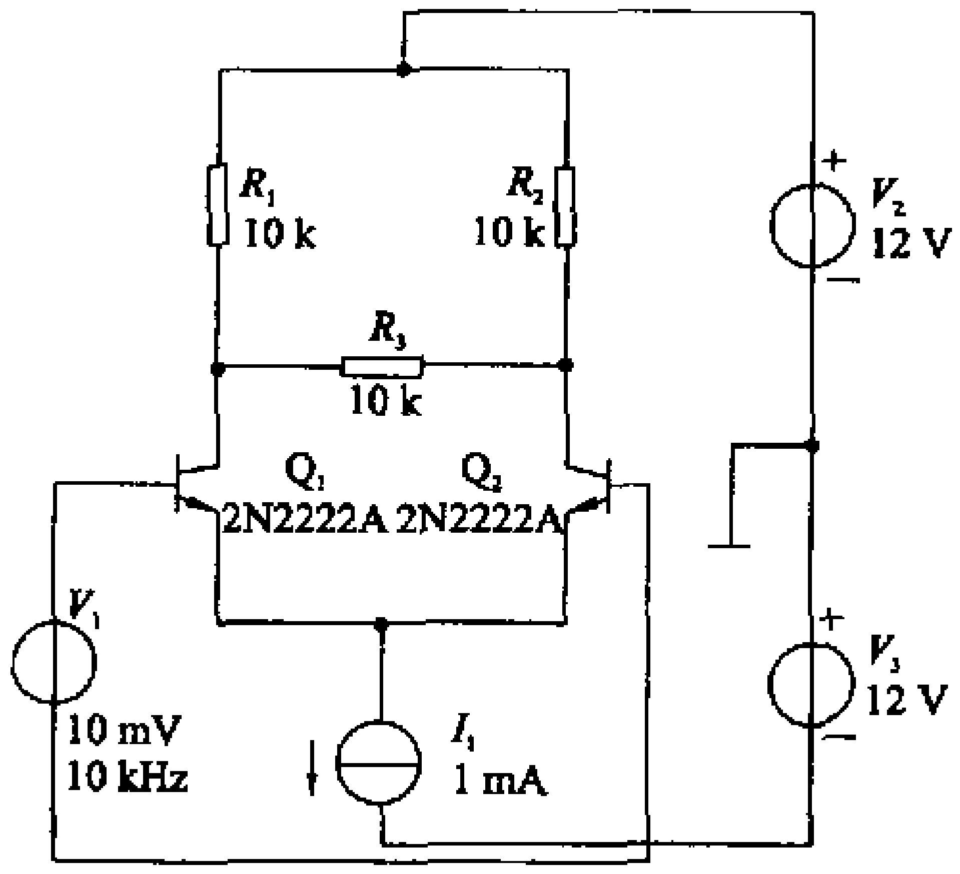
A common emitter amplifier with emitter resistor, whose feedback type is current serial negative feedback, is shown in Fig. 3(b).



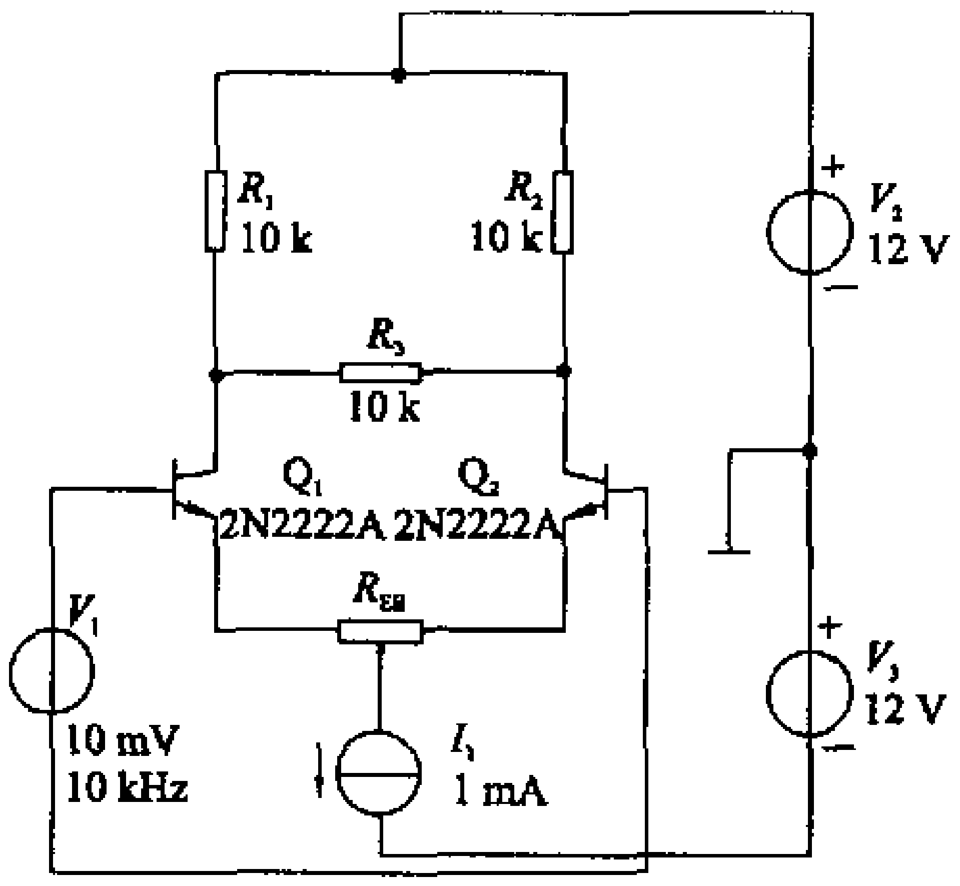
(a) Single bipolar transistor amplifier



(b) Single transistor amplifier with negative feedback



(c) Differential amplifier



(d) Differential amplifier with negative feedback

Fig. 3 Fundamental circuits

The model shown in Fig. 4(a) can be set up for a negative feedback amplifier, where k_f represents the feedback coefficient.

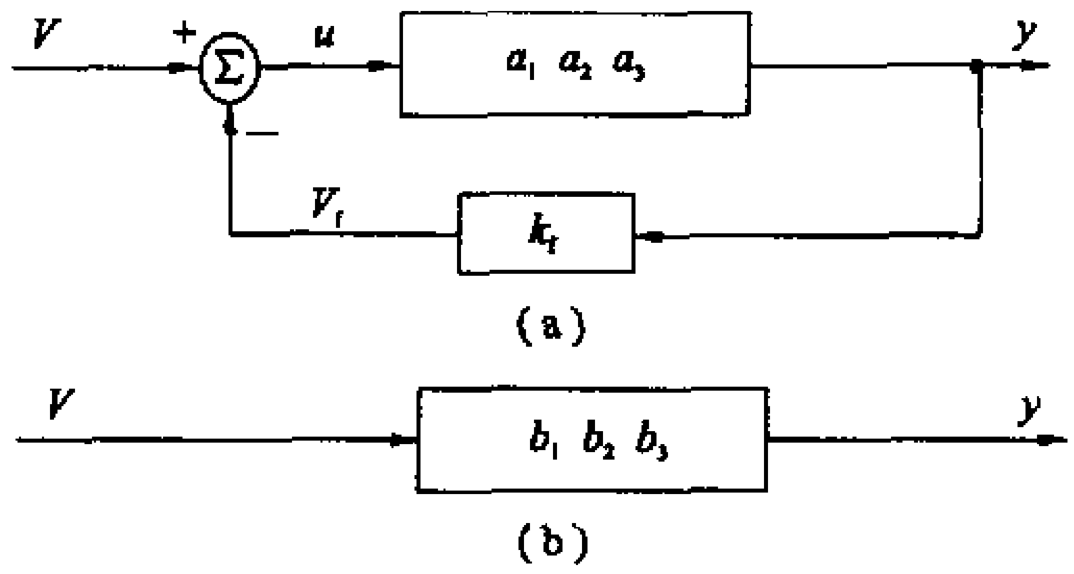


Fig. 4 Negative feedback amplifier model

From the model we obtain

$$y = a_1u + a_2u^2 + a_3u^3 \quad (20)$$

$$u = V - k_f y \quad (21)$$

$$y = b_1V + b_2V^2 + b_3V^3 \quad (22)$$

Substituting of Eqs. (21,22) into Eq. (20)

$$b_1 = \frac{a_1}{1 + a_1k_f} \quad (23)$$

$$b_2 = \frac{a_2}{(1 + a_1k_f)^3} \quad (24)$$

$$b_3 = \frac{a_3(1 + a_1k_f) - 2a_2^2k_f}{(1 + a_1k_f)^5} \quad (25)$$

According to Eq. (13) we get the intercept point IP_{3f} of negative feedback amplifier with emitter resistor R_E as

$$IP_{3f} = \sqrt{\frac{4}{3}\frac{a_1(1 + a_1k_f)^4}{a_3(1 + a_1k_f) - 2a_2^2k_f}} \quad (26)$$

in which

$$k_f = \frac{u_f}{i_o} = R_E, a_1 = \frac{I_{CQ}}{U_T}$$

$$a_2 = \frac{1}{2\,!}\frac{I_{CQ}}{U_T^2}, a_3 = \frac{1}{3\,!}\frac{I_{CQ}}{U_T^3} \quad (27)$$

Provided the parameters of Q_1, Q_2 in the differential pair are strictly symmetrical, we obtain the relation between output current i_o and input voltage u_{id} as

$$i_o = i_{C1} - i_{C2} = I_{EE}\mathrm{th}(\frac{u_{id}}{2U_T}) \quad (28)$$

Under small signal u_{id} condition

$$\mathrm{th}(x) \approx x - \frac{x^3}{3} \quad (29)$$

Eq. (28) can be expanded as

$$i_o = I_{EE}\left[\frac{u_{id}}{2U_T} - \frac{1}{3}\left(\frac{u_{id}}{2U_T}\right)^3\right] \quad (30)$$

It can be obtained from Eq. (13) that

$$IP_3 = 4U_T \quad (31)$$

Fig. 3 (d) shows the differential amplifier with emitter resistor R_{EE}, in which Q_1 and Q_2 are strictly symmetrical. This is a current serial negative feedback amplifier with R_{EE} as negative resistor and $k_f = R_{EE}$ as feedback coefficient. From Eq. (26), in which $a_1 = \frac{I_{EE}}{2U_T}$, $a_2 = 0$, $a_3 = \frac{I_{EE}}{3}\left(\frac{1}{2U_T}\right)^3$, IP_{3f} can be obtained as

$$IP_{3f} = 4U_T\sqrt{(1 + \frac{I_{EE}}{2U_T}R_{EE})^3} \quad (32)$$

3 SIMULATION RESULTS

By software Multisim the circuits in Fig. 3(a ~ d) are simulated with bipolar transistor 2N2222A, whose parameters are shown in Table 1.

Table 1 Parameters of 2N2222A

Forward transfer coefficient	Reverse transfer coefficient	Saturation current/A
220	4	3.0611E−14
Base resistance /kΩ	Emitter resistance/kΩ	Collector resistance/kΩ
0.13	0.22	0.12

With regards to Fig. 3(b), R_E is among the range of 0~1.5 kΩ. The larger R_E is, the deeper the negative feedback of amplifier is. The circuit in Fig. 3(a) is just under the condition that $R_E = 0$ in Fig. 3(b). Comparison of theoretical analyses with simulational results is shown in Fig. 5.

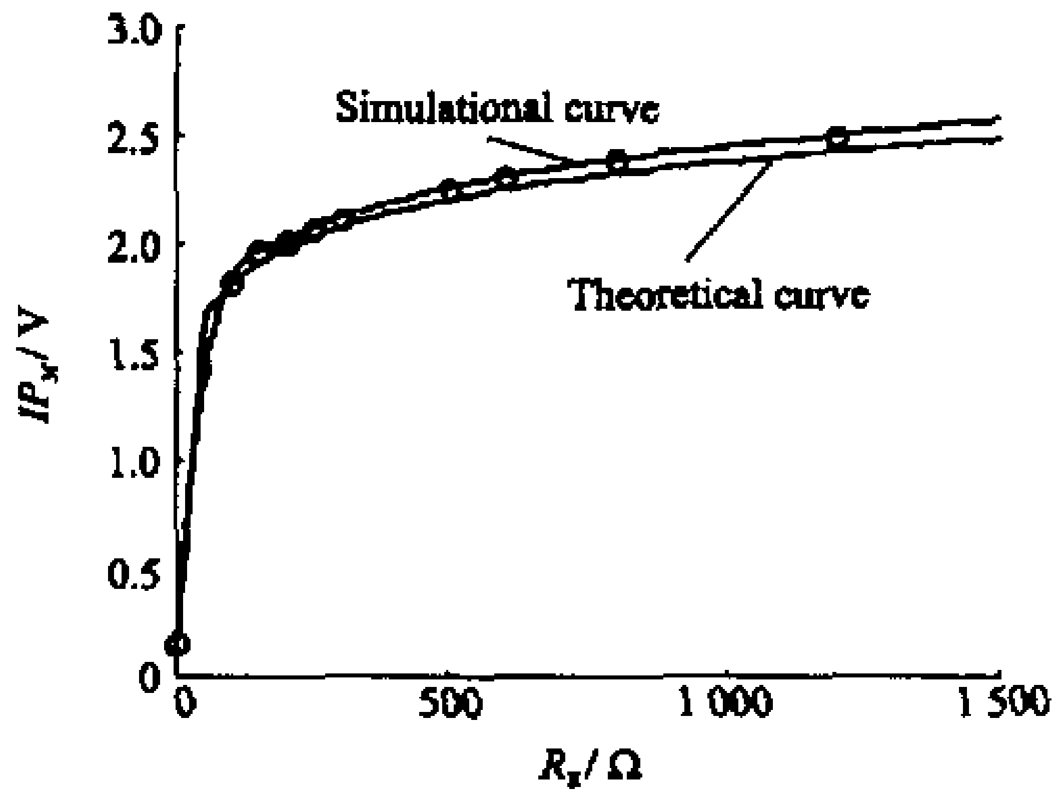


Fig. 5 Effect of R_E to IP_3 in single transistor amplifier

It is the same to the circuits shown in Fig. 3 (c),(d). Fig. 3(c) is also a special case of Fig. 3 (d) when $R_{EE} = 0$. Comparison is shown in Fig. 6.

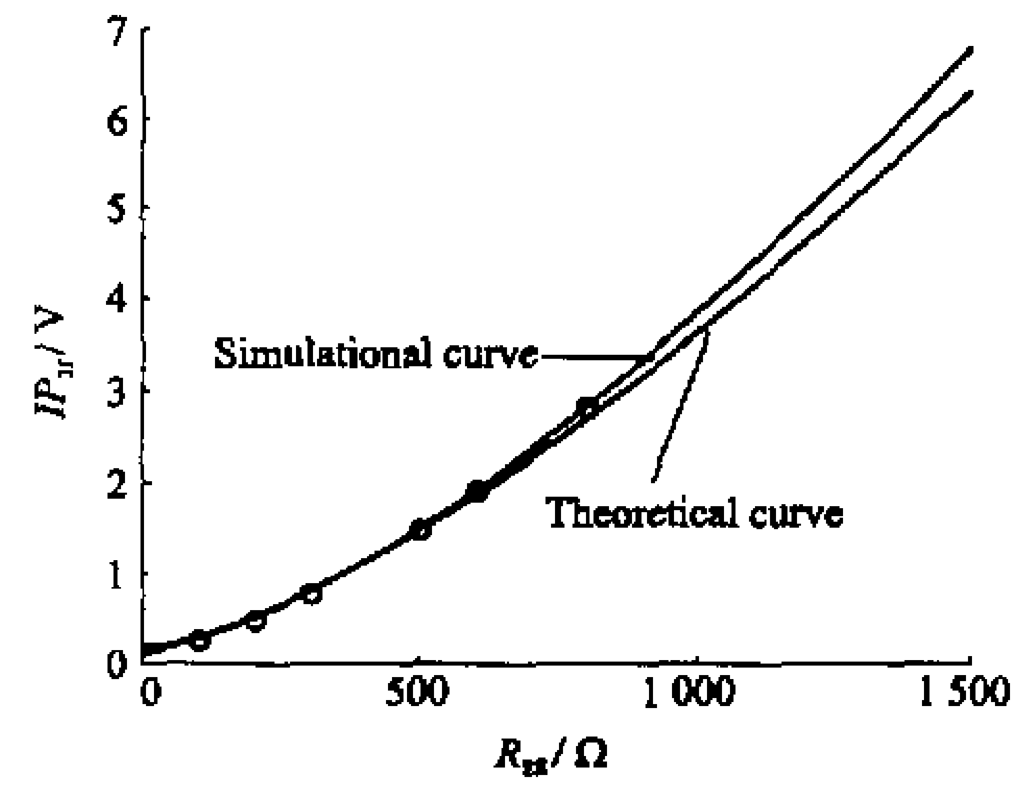


Fig. 6 Effect of R_{EE} to IP_3 in differential amplifier

4 CONCLUSION

In this paper the third-order Intercept point (IP_3) is taken to analyze distortion in transistor amplifier. The larger the IP_3, the better the linear performance of the amplifier. With negative feedback the linear characteristic can be improved a lot, which is proved by the congruity between the theoretical analyses and simulational results.

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放大器中非线性失真的研究

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摘要　晶体管是非线性器件,在放大信号时会产生非线性失真。针对弱非线性失真,推导了总谐波人(THD)、二阶互调失真(IM_2)、三阶互调失真(IM_3)和截点(IP_3)的表达式,借助于 Multisim 软件,对晶体管共发放大器、发射极带反馈电阻的失发放大器、差分放大器和发射极间带反馈电阻的差分放大器进行了仿真,并与理论结果作了比较,结果令人满意。

关键词:非线性失真;互调失真;截点;负反馈;差分对