
Inventory - Prerequisite

Economic Order Quantity

Outline

- ◆ Why to hold cycle inventories?
- ◆ Economies of scale to reduce fixed costs per unit.

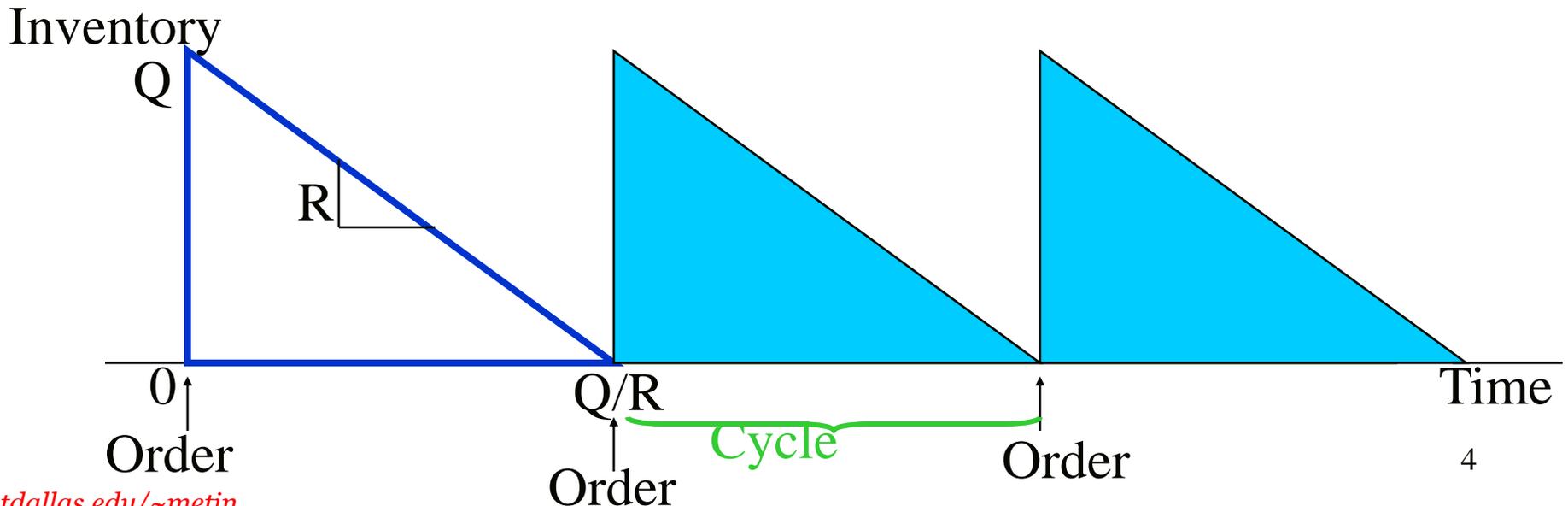
Role of Inventory in the Supply Chain

- ◆ **Overstocking: Amount available exceeds demand**
 - Liquidation, Obsolescence, Holding
- ◆ **Understocking: Demand exceeds amount available**
 - Lost margin
 - Future sales
 - » Consistent understocking reduces the customer demand

Goal: Matching supply and demand

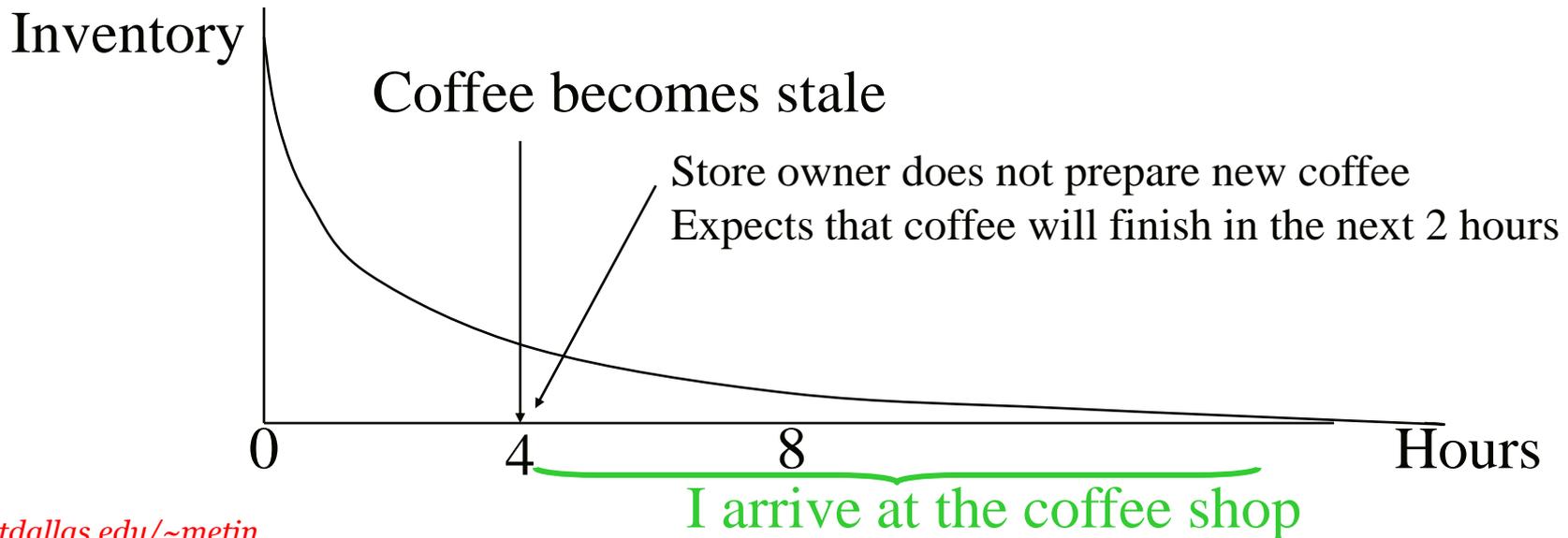
Batch or Lot size

- ◆ **Batch = Lot** = quantity of products bought / produced together
 - But not simultaneously, since most production can not be simultaneous
 - **Q**: Lot size. **R**: Demand per time, the book uses **D** for **R**.
- ◆ Consider sales at a Jean's retailer with demand of 10 jeans per day and an order size of 100 jeans.
 - $Q=100$. $R=10/\text{day}$.



Demand affected by visibility

- ◆ Demand is higher when the inventory is higher and is smaller when the inventory is smaller.
 - When I am buying coffee, it is often not fresh. Why?
 - Fresh coffee is consumed fast but stale coffee is not.



Batch or Lot size

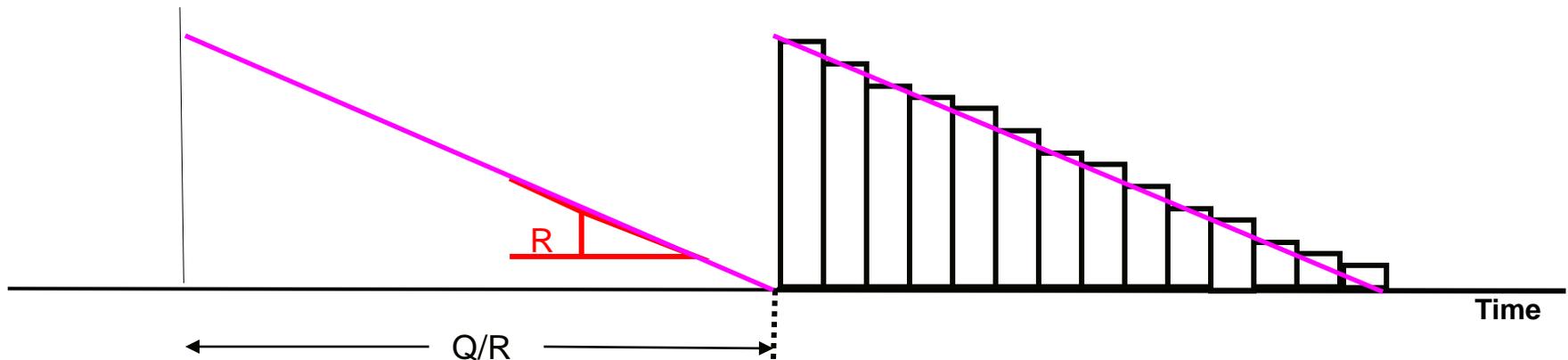
- ◆ **Cycle inventory**=Average inventory held during the cycle
= $Q/2=50$ jean pairs
- ◆ Average flow time
 - Remember Little's law
 - = (Average inventory)/(Average flow rate) = $(Q/2)/R=5$ days
 - Some jeans stay in the inventory for 10 days, some for 0 day.
- ◆ Long flow times make a company vulnerable in the case of product / technology changes
- ◆ Lower cycle inventory decreases working (operating) capital needs and space requirements for inventory
- ◆ Then, **why not to set Q as low** as possible?

Why to order in (large) lots?

- ◆ Fixed ordering cost: S
 - Increase the lot size to decrease the fixed ordering cost per unit
- ◆ Material cost per unit: C
- ◆ Holding cost: Cost of carrying 1 unit in the inventory: H
 - $H:=h.C$
 - h : carrying \$1 in the inventory $>$ internal rate of return $>$ interest rate
- ◆ Lot size is chosen by trading off holding costs against fixed ordering costs (and sometimes material costs).
 - Ex: Where to buy groceries from:

	Fixed cost (driving)	Material cost
Convenience store	low	HIGH
Sam's club	HIGH	low

Inventory held in N cycles



- ◆ Inventory held in a single cycle is approximately the sum of the bars on the right-hand side.
- ◆ These bars approximate the area of the triangle: $(1/2)(Q)(Q/R)$.
 - The base of the triangle Q/R
 - The height of the triangle Q
- ◆ Inventory held in N cycles is $N(1/2)(Q)(Q/R)$
- ◆ Length of N cycles $N(Q/R)$
- ◆ Inventory held per time: $(1/2)Q$

Economic Order Quantity - EOQ

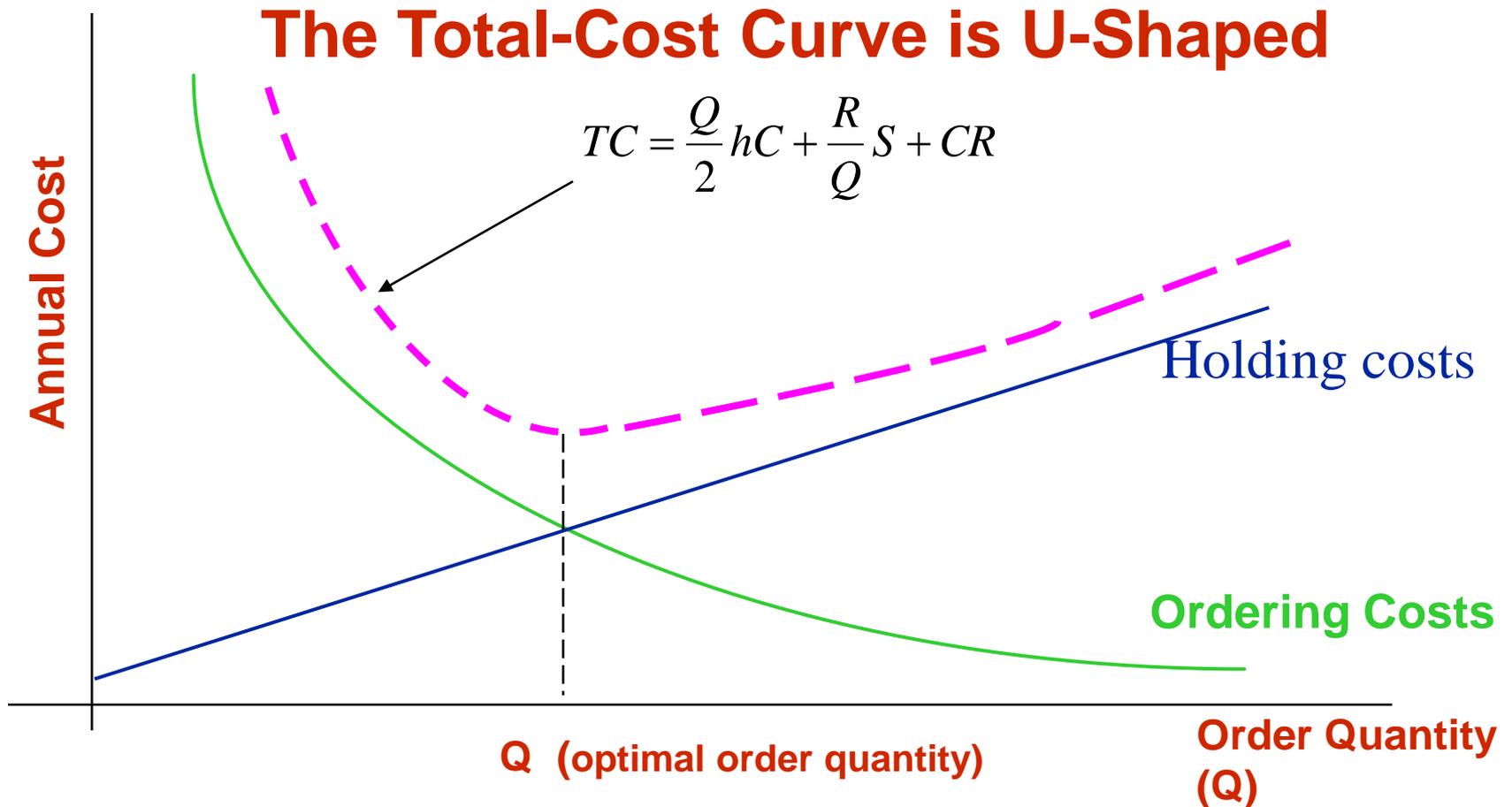
$$\text{TC} = \text{Annual carrying cost} + \text{Annual ordering cost} + \text{Purchasing cost}$$

$$\text{TC} = \frac{Q}{2} hC + \frac{R}{Q} S + CR$$

Total cost is simple function of the lot size Q .

Note that we can drop the last term, it is not affected by the choice of Q .

Cost Minimization Goal



Deriving the EOQ

Take the derivative of the total cost function and set the derivative equal to zero to solve for Q. Total cost curve is convex i.e. curvature is upward so we obtain the minimizer.

$$EOQ = \sqrt{\frac{2RS}{hC}} \quad T = \frac{EOQ}{R} = \sqrt{\frac{2S}{RhC}} \quad n = \frac{R}{EOQ} = \sqrt{\frac{RhC}{2S}}$$

T: Reorder interval (cycle) length = **EOQ/R**.

n: Ordering frequency: number of orders per unit time = **R/EOQ**.

The total cost (without purchasing cost) curve reaches its minimum where the inventory carrying and ordering costs are equal.

$$\text{Total cost}(Q = EOQ) = \sqrt{2RShC}$$

EOQ example

**Demand, $R = 12,000$ computers per year. Unit cost, $C = \$500$
Holding cost, $h = 0.2$. Fixed cost, $S = \$4,000/\text{order}$.
Find EOQ, Cycle Inventory, Average Flow Time, Optimal
Reorder Interval and Optimal Ordering Frequency.**

EOQ = 979.79, say 980 computers

Cycle inventory = $\text{EOQ}/2 = 490$ units

Average Flow Time = $\text{EOQ}/(2R) = 0.49$ month

Optimal Reorder interval, $T = 0.0816$ year = 0.98 month

Optimal ordering frequency, $n=12.24$ orders per year.

Key Points from Batching

- ◆ In deciding the optimal lot size the **trade off** is between setup (order) cost and holding cost.
- ◆ If **demand increases** by a factor of 4, it is optimal to increase batch size by a factor of 2 and produce (order) twice as often. Cycle inventory (in units) doubles. *Cycle inventory (in days of demand) halves.*
- ◆ If lot size is to be reduced, one has to reduce fixed order cost. To reduce lot size by a factor of 2, **fixed ordering cost** has to be reduced by a factor of 4. This is what JIT strives to do.

Strategies for reducing fixed costs

- ◆ In production

- Standardization / dedicated
- Simplification
- Set up out of the production line
 - » **Service:** At Taiwanese restaurants food order is taken from the customer while customers are waiting for a table.
 - » **Manufacturing:** Toyota die change in stamping operation

Setup Time (Cost) Reduction

- ◆ Set up time has two components
 - External set up: Executed while the machine is operating
 - Internal setup: Executed while the machine is stopped.

EX: Consider the setup for a lecture:

- » Erase the board, bring the screen down, turn on laptop, project to screen
- » Turning on the laptop is the bottleneck
 - ◆ Which operations are external/internal w.r.t. turning on the laptop?

EX: Roplast industries (a manufacturer of plastic bags) reduced setup times by 68%, down to 23 mins, and targeting 15 mins. This allowed Roplast run smaller batches.



EX: 1000 ton metal stamp
Used in making automobile body
SMED: Single minute exchange of a die

More examples of Personal External setups

- ◆ Announcing hw questions on the course web page increases the time available for the lecture.
- ◆ At the Java coffee store (1st floor of SOM), insulators are put on one coffee cup of each size before the customers order coffee.
- ◆ I have investigated the idea of not removing belts from my trousers to reduce the time I take to dress up in the morning.

Strategies for reducing fixed costs in delivery

In delivery

- Third party logistics
- Aggregating multiple products in a single order
 - » Temporal, geographic aggregation
- Various truck sizes, difficult to manage

Summary

- ◆ Optimal order quantity
- ◆ Reducing fixed costs per unit