Due: Wed, April 14, 2004

1. There are production processes for which the difference between the cost of producing the maximum number of units allowed by some capacity restriction and the cost of producing any number of units less than this maximum is negligible, i.e., ordering is by batches. Consider a one-stage model where the only two costs are holding costs given by
   \[ h(y - D) = (3/10)(y - D) \]
   and the penalty cost for unsatisfied demand given by
   \[ p(D - y) = (3/2)(D - y) \].

   The density function for demand is given by
   \[
   \begin{cases}
   \frac{e^{-\xi/25}}{25}, & \text{for } \xi \geq 0 \\
   0, & \text{otherwise.}
   \end{cases}
   \]

   If you order, you must order in batches of 50 units of product, and this quantity is delivered instantaneously. Thus, if \( x \) denotes the quantity on hand, and you do not order, then \( y = x \). If you order one batch, then \( y = x + 50 \). Let \( G(y) \) denote the total expected cost of this inventory problem when there are \( y \) units available for the period (after you have ordered).

   a) Write down the expression for \( G(y) \).
   b) What is the optimal ordering policy?

2. Find the optimal inventory policy for the following two-period model. Let the density of the demand \( D \) be given by
   \[
   \varphi(\xi) = \begin{cases} 
   \frac{1}{25} e^{-\xi/25}, & \text{for } \xi \geq 0 \\
   0, & \text{otherwise,}
   \end{cases}
   \]
   and the costs are:
   - holding = $0.30 per item,
   - shortage = $1.50 per item,
   - purchase price = $1 per item.

3. Problem 2.2 in the text. Show your work.