This is a practice problem set for the final exam. Study these problems carefully!

**Problem F.1** Fourier Transform Properties

First evaluate the Fourier transform of a continuous-time signal \( x(t) = e^{-bt}u(t) \) where \( b \) is a positive constant. Then, determine the Fourier transform \( V(jw) \) of the following signals.

(a) \( v(t) = x(5t - 4) \)
(b) \( v(t) = t^2x(t) \)
(c) \( v(t) = x(t)e^{jt} \)
(d) \( v(t) = x(t)\cos 4t \)
(e) \( v(t) = \frac{d^2x(t)}{dt^2} \)
(f) \( v(t) = x(t) * x(t) \)
(g) \( v(t) = \frac{1}{jt-b} \) Use duality property.

**Problem F.2** Inverse Fourier Transform

Compute the inverse Fourier transform of the following functions

(a) \( X(jw) = 3\delta(w - 1) + j2\delta(w - 2) + 3\delta(w + 1) - j2\delta(w + 2) \)
(b) \( X(jw) = \cos 4w \)
(c) \( X(jw) = \frac{8nw}{jw^2 + 2}e^{-jw^2} \)
(d) \( X(jw) = \frac{3(3+jw)}{(4+jw)(2+jw)} \)

**Problem F.3** Fourier Transform

By first expressing \( x(t) \) in terms of rectangular pulse functions, compute the Fourier transform of the signals below.
Problem F.4 Convolution Property of Fourier Transform and Filtering
Consider an LTI system with frequency response $H(jw)$ shown below

Determine the system response $y(t)$ to the input $x(t) = 2 \cos w_0 t + 3 \sin 2w_0 t + 4 \cos 4w_0 t$ where $2w_0 < w_1 < 4w_0$.

Problem F.5
The frequency which, under the sampling theorem, must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals.

(a) $x(t) = 1 + \cos 2,000\pi t + \sin 4,000\pi t$

(b) $X(jw) = \begin{cases} 
1 & \text{for } |w| \leq 1000\pi \\
0 & \text{otherwise} 
\end{cases}$