Example for the use of Distributive Property

\[ x[n] = \left( \frac{1}{3} \right)^n u[n] + 3^n u[-n-1] \]

\[ h[n] = u[n+3] \]

\[ y[n] = x[n] * h[n] \]

\[ x[n] = x_1[n] + x_2[n] \]

\[ y[n] = \{ x_1[n] + x_2[n] \} * h[n] \]

\[ y[n] = x_1[n] * h[n] + x_2[n] * h[n] \]

\[ y[n] = y_1[n] + y_2[n] \]

\[ y_1[n] = \sum_{k=0}^{n+3} \left( \frac{1}{3} \right)^n \quad n + 3 \geq 0 \]

\[ y_1[n] = 0 \quad n + 3 < 0 \]

\[ y_2[n] = \sum_{k=0}^{n} \left( \frac{1}{3} \right)^n \quad n \geq -3 \]
\[ y_1[n] = \frac{1 - \left(\frac{1}{3}\right)^{n+4}}{2/3} = \frac{3}{2} \left(1 - \left(\frac{1}{3}\right)^{n+4}\right) u[n+3] \]

\[ y_2[n] = \sum_{k=-\infty}^{\infty} 3^k \]

\[ y_2[n] = \sum_{k=-n-3}^{\infty} \left(\frac{1}{3}\right)^k = \sum_{l=0}^{\infty} \left(\frac{1}{3}\right)^{l-n-3} = \left(\frac{1}{3}\right)^{-n-3} \times \frac{1}{2/3} = \frac{3}{2} \left(\frac{1}{3}\right)^{-n-3} = \frac{3}{2} 3^{n+3} \]

where \( l = k + n + 3 \)

\[ y_2[n] = \frac{3}{2} 3^{n+3} u[-n-4] + \frac{1}{2} u[n+3] \]

\[ y_1[n] + y_2[n] \]

\[ = \left(2 - \frac{3}{2} \left(\frac{1}{3}\right)^{n+4}\right) u[n+3] + \frac{3}{2} 3^{n+3} u[n-4] \]
The Associative Property

Another important and useful property of convolution is associative property.

In D-T: \[ x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n] \]

In C-T: \[ x[t] * (h_1[t] * h_2[t]) = (x[t] * h_1[t]) * h_2[t] \]

Consequences of the associative property

\[ y[n] = x[n] * h_1[n] * h_2[n] \]

\[ y[t] = x[t] * h_1[t] * h_2[t] \]

That is, it does not matter in which order we convolve these signals.
The Associative Property

An interpretation of the associative property is as follows:

Series connection of the two systems is equivalent to

Commutative property

Order of the systems can be thus changed.
Conclusion

Impulse response of the cascade of two LTI systems is the convolution of their individual responses.

It does not depend on the order in which they are cascaded.

Being able to interchange the order of the systems in a cascade is particular to LTI systems. It does not apply to non-linear systems in general.
Example (p 2-24)

\[ h_2[n] = \delta[n] - \delta[n-1] \]

\[ h[n] = h_1[n] * h_2[n] * h_2[n] \]

This is the overall impulse response.

\[ h_2[n] * h_1[n] = \delta[n] - 2\delta[n-1] + \delta[n-2] \]

\[ x[n] * \delta[n-a] = x[n-a] \quad \text{this is a delay function.} \]

\[ \delta[n-b] * \delta[n-a] = \delta[n-b-a] \]

\[ \delta[n] * \delta[n] = \delta[n] \]