In Neil Salkind (Ed.), *Encyclopedia of Research Design.*
Thousand Oaks, CA: Sage. 2010

Fisher's Least Significant Difference (LSD) Test

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1 Overview

When an analysis of variance (ANOVA) gives a significant result, this indicates that at least one group differs from the other groups. Yet, the omnibus test does not indicate which group differs. In order to analyze the pattern of difference between means, the ANOVA is often followed by specific comparisons, and the most commonly used involves comparing two means (the so called "pairwise comparisons").

The first pairwise comparison technique was developed by Fisher in 1935 and is called the *least significant difference* (LSD) test. This technique can be used only if the ANOVA F omnibus is significant. The main idea of the LSD is to compute the smallest significant difference (*i.e.,* the LSD) between two means as if these means had been the only means to be compared (*i.e.,* with a t test) and to declare significant any difference larger than the LSD.

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2 Notations

The data to be analyzed comprise A groups, a given group is denoted a. The number of observations of the a-th group is denoted S_a. If all groups have the same size it is denoted S. The total number of observations is denoted N. The mean of Group a is denoted M_{a+}. From the ANOVA, the mean square of error (*i.e.,* within group) is denoted $MS_{S(A)}$ and the mean square of effect (*i.e.,* between group) is denoted MS_A.

3 Least significant difference

The rationale behind the LSD technique value comes from the observation that, when the null hypothesis is true, the value of the t statistics evaluating the difference between Groups a and a' is equal to

$$t = \frac{M_{a+} - M_{a'+}}{\sqrt{MS_{S(A)}\left(\frac{1}{S_a} + \frac{1}{S_{a'}}\right)}} \ , \tag{1}$$

and follows a student's t distribution with $N - A$ degrees of freedom. The ratio t would therefore be declared significant at a given α level if the value of t is larger than the critical value for the α level obtained from the t distribution and denoted $t_{\nu,\alpha}$ (where $\nu = N - A$ is the number of degrees of freedom of the error, this value can be obtained from a standard t table). Rewriting this ratio shows that, a difference between the means of Group a and a' will be significant if

$$|M_{a+} - M_{a'+}| > \text{LSD} = t_{\nu,\alpha}\sqrt{MS_{S(A)}\left(\frac{1}{S_a} + \frac{1}{S_{a'}}\right)} \tag{2}$$

When there is an equal number of observation per group, Equation 2 can be simplified as:

$$\text{LSD} = t_{\nu,\alpha}\sqrt{MS_{S(A)}\frac{2}{S}} \tag{3}$$

In order to evaluate the difference between the means of Groups a and a', we take the absolute value of the difference between the means and compare it to the value of LSD. If

$$|M_{i+} - M_{j+}| \geq \text{LSD} \ , \tag{4}$$

then the comparison is declared significant at the chosen α-level (usually .05 or .01). Then this procedure is repeated for all $\frac{A(A-1)}{2}$ comparisons.

Note that LSD has more power compared to other post-hoc comparison methods (*e.g.,* the honestly significant difference test, or Tukey test) because the α level for each comparison is not corrected for multiple comparisons. And, because LSD does not correct for multiple comparisons, it severely inflates Type I error (*i.e.,* finding a difference when it does not actually exist). As a consequence, a revised version of the LSD test has been proposed by Hayter (and is knows as the *Fisher-Hayter* procedure) where the modified LSD (MLSD) is used instead of the LSD. The MLSD is computed using the Studentized range distribution q as

$$\text{MLSD} = q_{\alpha,A-1}\sqrt{\frac{MS_{S(A)}}{S}} \ . \tag{5}$$

where $q_{\alpha,A-1}$ is the α level critical value of the Studentized range distribution for a range of $A-1$ and for $\nu = N - A$ degrees of freedom. The MLSD procedure is more conservative than the LSD, but more powerful than the Tukey approach because the critical value for the Tukey approach is obtained from a Studentized range distribution equal to A. This difference in range makes Tukey's critical value always larger than the one used for the MLSD and therefore it makes Tukey's approach more conservative.

4 Example

In a series of experiments on eyewitness testimony, Elizabeth Loftus wanted to show that the wording of a question influenced witnesses' reports. She showed participants a film of a car accident, then asked them a series of questions. Among the questions was one of five versions of a critical question asking about the speed the vehicles were traveling:

1. How fast were the cars going when they *hit* each other?
2. How fast were the cars going when they *smashed into* each other?
3. How fast were the cars going when they *collided with* each other?
4. How fast were the cars going when they *bumped* each other?
5. How fast were the cars going when they *contacted* each other?

The data from a fictitious replication of Loftus' experiment are shown in Table 1. We have $A = 4$ groups and $S = 10$ participants *per* group.

The ANOVA found an effect of the verb used on participants' responses. The ANOVA table is shown in Table 2.

Table 1 Results for a fictitious replication of Loftus & Palmer (1974) in miles per hour

	Contact	Hit	Bump	Collide	Smash
	21	23	35	44	39
	20	30	35	40	44
	26	34	52	33	51
	46	51	29	45	47
	35	20	54	45	50
	13	38	32	30	45
	41	34	30	46	39
	30	44	42	34	51
	42	41	50	49	39
	26	35	21	44	55
$M_{\cdot +}$	30	35	38	41	46

Table 2 ANOVA results for the replication of Loftus and Palmer (1974).

Source	*df*	*SS*	*MS*	*F*	*Pr*(*F*)
Between: A	4	1,460.00	365.00	4.56	.0036
Error: $S(A)$	45	3,600.00	80.00		
Total	49	5,060.00			

4.1 LSD

For an α level of .05, the LSD for these data is computed as:

$$\begin{aligned} \text{LSD} &= t_{\nu,.05}\sqrt{MS_{S(A)}\frac{2}{n}} \\ &= t_{\nu,.05}\sqrt{80.00 \times \frac{2}{10}} \\ &= 2.01\sqrt{\frac{160}{10}} \\ &= 2.01 \times 4 \\ &= 8.04\ . \end{aligned} \tag{6}$$

A similar computation will show that, for these data, the LSD for an α level of .01, is equal to $\text{LSD} = 2.69 \times 4 = 10.76$.

For example, the difference between $M_{\text{CONTACT}+}$ and $M_{\text{HIT}+}$ is declared non significant because

$$|M_{\text{CONTACT}+} - M_{\text{HIT}+}| = |30 - 35| = 5 < 8.04\ . \tag{7}$$

The differences and significance of all pairwise comparisons are shown in Table 3.

Table 3 LSD. Difference between means and significance of pairwise comparisions from the (fictitious) replication of Loftus and Palmer (1974). Differences larger than 8.04 are significant at the $\alpha = .05$ level and are indicated with *, differences larger than 10.76 are significant at the $\alpha = .01$ level and are indicated with **.

	Experimental Group				
	$M_{1.+}$ Contact 30	$M_{2.+}$ Hit 1 35	$M_{3.+}$ Bump 38	$M_{4.+}$ Collide 41	$M_{5.+}$ Smash 46
$M_{1.+} = 30$ Contact	0.00	5.00 *ns*	8.00 *ns*	11.00**	16.00**
$M_{2.+} = 35$ Hit		0.00	3.00 *ns*	6.00 *ns*	11.00**
$M_{3.+} = 38$ Bump			0.00	3.00 *ns*	8.00 *ns*
$M_{4.+} = 41$ Collide				0.00	5.00 *ns*
$M_{5.+} = 46$ Smash					0.00

Table 4 MLSD. Difference between means and significance of pairwise comparisions from the (fictitious) replication of Loftus and Palmer (1974). Differences larger than 10.66 are significant at the $\alpha = .05$ level and are indicated with *, differences larger than 13.21 are significant at the $\alpha = .01$ level and are indicated with **.

	Experimental Group				
	$M_{1.+}$ Contact 30	$M_{2.+}$ Hit 1 35	$M_{3.+}$ Bump 38	$M_{4.+}$ Collide 41	$M_{5.+}$ Smash 46
$M_{1.+} = 30$ Contact	0.00	5.00 *ns*	8.00 *ns*	11.00*	16.00**
$M_{2.+} = 35$ Hit		0.00	3.00 *ns*	6.00 *ns*	11.00*
$M_{3.+} = 38$ Bump			0.00	3.00 *ns*	8.00 *ns*
$M_{4.+} = 41$ Collide				0.00	5.00 *ns*
$M_{5.+} = 46$ Smash					0.00

4.2 MLSD

For an α level of .05, the value of $q_{.05,A-1}$ is equal to 3.77 and the MLSD for these data is computed as:

$$\text{MLSD} = q_{\alpha,A-1}\sqrt{\frac{MS_{S(A)}}{S}} = 3.77 \times \sqrt{8} = 10.66 \; . \tag{8}$$

The value of $q_{.01,A-1} = 4.67$, and a similar computation will show that, for these data, the MLSD for an α level of .01, is equal to $\text{MLSD} = 4.67 \times \sqrt{8} = 13.21$..

For example, the difference between $M_{\text{CONTACT+}}$ and $M_{\text{HIT+}}$ is declared non significant because

$$|M_{\text{CONTACT+}} - M_{\text{HIT+}}| = |30 - 35| = 5 < 10.66 \; . \tag{9}$$

The differences and significance of all pairwise comparisons are shown in Table 4.

Related entries

Analysis of variance, Bonferroni procedure, Honestly significant difference (HSD) test, Multiple comparison test, Newman-Keuls test, Pairwise comparisons, Post-hoc comparisons, Scheffe's test, Tukey test.

Further readings

Abdi, H., Edelman, B., Valentin, D., & Dowling, W.J. (2009). *Experimental Design and Analysis for Psychology*. Oxford: Oxford University Press.

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